Convex Vacancy Creation Costs and On-the-Job Search in a Global Economy*

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Abstract

We combine convex vacancy creation costs and on-the-job search in a new trade theory model with heterogeneous firms to explain how trade liberalization affects trade patterns, wages, and wage inequality. To obtain a realistic trade pattern, we need convex vacancy creation costs; to obtain a realistic wage distribution, we need on-the-job search. To get an effect of trade liberalization on wage inequality, we need both because with linear vacancy creation costs trade liberalization affects all wages in equal proportion. Furthermore, with sufficiently high convex vacancy creation costs, not all firms export to all foreign markets even if trade is fully liberalized. This implies that wage inequality under free trade is always higher than under autarky.

Keywords: On-the-job search; convex vacancy creation costs; international trade; heterogeneous firms; monopolistic competition

JEL-Codes: F16, F12, J64, L11


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1 Introduction

In response to the public’s growing concern that trade liberalization adversely affects unemployment and wage inequality, researchers have begun to integrate labor market models that generate unemployment and wage distributions into classical trade models. These models predict positive as well as negative effects of trade liberalization on unemployment and find an inverse u-shape relationship between trade liberalization and residual wage dispersion among workers with similar observed characteristics.  

The existing theoretical trade models, which are used to investigate the effect of trade liberalization on residual wage inequality (i.e., wage inequality within an industry not explained by observables), distinguish only between non-exporting and exporting firms. They disregard the richness of firms’ export behavior including the facts that most exporting firms sell to only one foreign market, that the number of firms that sell to multiple markets declines with the number of destinations, and that the export strategies of observationally equivalent firms vary greatly across countries with similar characteristics. Our theoretical model suggests that the effect of trade liberalization on residual wage inequality depends on whether the underlying model generates only non-exporting and exporting firms or a trade pattern consistent with the empirical facts mentioned above.

We introduce convex vacancy creation costs and on-the-job search like in Burdett and Mortensen (1998) into a Melitz (2003)-type new trade model with monopolistic competition and heterogeneous firms. This leads to a simple and analytically tractable model that is in line with many empirical facts about both the trade and labor market side. To obtain realistic trade patterns we need convex vacancy creation costs because firms with convex vacancy creation costs and fixed entry costs for export markets find it optimal to sell only to a subset of foreign markets. More productive firms sell to more markets. To obtain a realistic wage distribution, we need on-the-job search, since without on-the-job search all firms would pay the reservation wage. To get an effect of trade liberalization on residual wage inequality we need both.

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1Section 2 presents a literature review, which contains the respective studies.
2Akerman, Helpman, Itskhoji, Muendler, and Redding (2013) provide nice empirical evidence for the importance of residual within-sector wage inequality and the role of the export status for wage payments.
4If we talk about wage inequality in the paper, we always mean residual wage inequality.
While we could have assumed other types of convex costs to obtain similar results for the trade pattern, we assume convex vacancy creation costs because they are well in line with the empirical evidence on the shape of the vacancy creation cost function. Manning (2006) and Coşar, Guner, and Tybout (2010) find direct evidence for a convex vacancy creation cost function using U.K. and Colombian data, respectively. Yashiv (2006) and Merz and Yashiv (2007) find a convex hiring cost function using U.S. data, and Blatter, Muehlemann, and Schenker (2012) find the same using Swiss data.

The result that trade liberalization affects residual wage inequality only in a model with convex vacancy creation costs and on-the-job search follows from the fact that with linear vacancy creation costs firms post vacancies until marginal profits are equal to the constant cost of vacancy creation. This implies that all firms, irrespective of their export strategy, have the same marginal profit in equilibrium. As in Burdett and Mortensen (1998), more productive firms pay higher wages and employ more workers, but wages paid in equilibrium will not depend on a firm’s export strategy. Wages only depend on aggregate labor market tightness. Trade liberalization, which increases vacancy creation, therefore increases all wages proportionally, but leaves wage inequality unchanged, since all commonly used wage inequality measures (e.g., Gini coefficient) are invariant to proportional changes. With convex vacancy creation costs, marginal profits – and thus wages – depend on the number of destinations to which a firm exports. Wage inequality therefore depends on the openness of an economy.

If vacancy creation costs are sufficiently convex, then even if trade is fully liberalized, that is, if there are no additional costs for exporting, not all firms will export. The trade pattern mentioned above continues to persist. With linear vacancy creation costs, free trade implies like in Melitz (2003) and Helpman, Itskhoki, and Redding (2008, 2010) that all active firms in the economy serve the domestic and all export markets. This difference in trade patterns has implications for the relationship between trade liberalization and wage inequality. Starting from autarky, wage inequality first increases with trade liberalization as those firms that start to export increase their wages overproportionally. As the number of exporting firms increases, wage inequality decreases at the upper end of the wage distribution, that is, among high productive firms, since they profit equally from exporting. If all firms export to all foreign countries, all firms increase their wages proportionally and wage inequality is the same as under autarky. This leads to an inverse u-shape relationship between trade liberalization
and wage inequality if firms face linear vacancy creation costs. If vacancy creation costs are sufficiently convex, not all firms export even if trade is fully liberalized. In an open economy wage inequality is therefore always higher than under autarky. Our theoretical model therefore suggests that studies employing structural estimation to evaluate the extent to which trade liberalization can explain the rise in wage inequality should not only capture firm level employment and wage patterns but also the richness of firms’ export behavior.

The paper is structured as follows. In the next section we provide a short overview of the related literature. In Section 3 we present the general framework that links a Melitz’s (2003)-type new trade theory model with the on-the-job search model by Burdett and Mortensen (1998). In Section 4 we present the equilibrium of our model, which we characterize and compare in Section 5 with the complementary setups that are based either on linear vacancy creation costs and/or on no on-the-job search. This section also contains our main results on the comparative statics of trade liberalization on trade patterns, wages, and wage inequality for the different frameworks. Section 6 concludes.

2 Related Literature

Researchers have begun to integrate labor market models that generate unemployment into classical trade models. Brecher (1974) was the first to study minimum wages in the Heckscher-Ohlin model with two countries, two factors, and two goods, and Davis (1998) generalized this model. Davidson, Martin, and Matusz (1999) and Davidson and Matusz (2004) introduced search frictions and wage bargaining into multi-sector models of international trade governed by comparative advantage and Helpman and Itskhoki (2010) and Felbermayr, Prat, and Schmerer (2011) did the same for the new trade theory based on the Melitz (2003) model. Cuñat and Melitz (2010, 2012) embed firing restrictions in a Ricardian setting. This variety of models predicts positive as well as negative effects of trade liberalization on unemployment. The empirical literature finds evidence for positive (Dutt, Mitra, and Ranjan 2009; Felbermayr, Prat and Schmerer 2011; Hasan, Mitra, Ranjan, and Ahsan 2012) as well as for hardly any or negative effects (Attanasio, Goldberg, and Pavcnik 2004; Menezes-Filho and Muendler 2011).

These models’ silence on how trade liberalization affects the wage distribution was the impetus for a new strand of literature. Helpman, Itskhoki, and Redding (2008, 2010) use a matching framework to explain wage differences across observationally equivalent workers
by the firm-level heterogeneity that arises if some firms hire only high productive workers while others hire all types of workers. Egger and Kreickemeier (2012) explain intra-group wage inequality among ex ante identical workers using a fair wage effort mechanism. Amiti and Davis (2012) also assume a fair wage constraint and investigate the effects of input and output tariffs. All these models find that trade liberalization increases the wage distribution of ex ante identical workers. In our context, wage inequality is not the result of exogenously given fair wage preferences or the result of monitoring or screening costs, but is instead the result of workers’ continuous search for better paid jobs, as introduced in Burdett and Mortensen (1998). Suverato (2014) and Stijepic (2015) consider interesting extensions; the first builds on an on-the-job search model with bargaining developed by Mortensen (2009) in order to characterize the transitional dynamics of the wage distribution in response to trade liberalization, and the second considers different skill groups in an on-the-job search model based on Holzner and Launov (2010) and shows that trade liberalization increases the wage gap between skilled and unskilled workers.

Another strand of the literature, spurred by the emergence of firm-level datasets, documents and explains firms’ export behavior. Arkolakis (2010) and Eaton, Kortum, and Kramarz (2011) document that most exporting firms sell only to one foreign market, that the number of firms that sell to multiple markets declines with the number of destinations, and that the export strategies of observationally equivalent firms vary greatly across countries with similar characteristics. To explain the first two empirical observations, Arkolakis (2010) and Eaton, Kortum, and Kramarz (2011) introduce not only market but also firm-specific heterogeneity in entry costs and market size into the Melitz (2003) model. Convex vacancy creation costs can also explain the third finding. Convex costs of vacancies and hence production imply that exporting firms do not find it optimal to adjust their output so as to serve all foreign markets. Thus, even if two export markets are similar and firms are indifferent between them, they might export to just one country and not the other, because they find it too costly to increase production to serve both countries. Fajgelbaum (2013) also uses an on-the-job search equilibrium model based on Postel-Vinay and Robin (2002) and argues that this trade pattern arises because young firms need time to grow. Other explanations of the observed trade pattern are based on the heterogeneity among firms arising from information frictions. For example, Chaney (2014) proposes a dynamic model in which firms only export to markets where they have a contact, and Morales, Sheu and Zahler (2015) argue that firms
are more likely to export to destinations that are close to previous export destinations. A
different approach is taken by Eaton, Kortum, and Kramarz (2015) who focus on firm-to-firm
trade and explain the trade pattern by heterogeneity among firms and search frictions.

3 Framework

3.1 Labor Market and Workers’ Search Strategy

The model has an infinite horizon, is set in continuous time, and concentrates on steady
states. The measure of firms \( M \) in the economy will be endogenously determined in the
product market. Firms have to decide on the wage \( w \) they offer, the number of export
destinations \( j \) they serve, and the number of vacancies \( v \) they create. Each vacancy is
assumed to be contacted by a worker at the endogenous rate \( \eta(M) \). To keep the model
analytically tractable, we assume that firms face zero vacancy creation costs for \( v \leq \overline{v} \) and
infinitely high costs for \( v > \overline{v} \). This implies that all firms create \( \overline{v} \) vacancies. Hence, the
total number of contacts made by firms is given by \( \eta(M) M \overline{v} \). To highlight the role of convex
vacancy creation costs, in section 5.1 we consider the case where firms can post vacancies
given the linear cost function \( cv \) like in Mortensen (2003).\(^6\)

Workers’ life-time is exponentially distributed with parameter \( \phi \). They are risk neutral
and, for the sake of simplicity but without loss of generality, do not discount the future. Since
we normalize the measure of workers to one, \( \phi \) also describes the in- and outflow of workers
in the labor market. Workers are either unemployed and receive an unemployment benefit \( z \)
or they are employed and receive a wage \( w \). Both unemployed and employed workers search
for a job with the same intensity. Following Burdett and Mortensen (1998), the probability
of a worker meeting a firm follows a Poisson process with rate \( \lambda \). Since aggregation requires
that the total number of firm contacts equals the total number of worker contacts, we obtain,

\[
\lambda = \eta(M) M \overline{v}.
\]

Since all workers, regardless of whether they are employed or unemployed, search with the
same intensity, we know from Burdett and Mortensen (1998) that unemployed workers’ reservation wage equals the unemployment benefit and that employed workers will accept any wage above their current wage. The wage offer distribution is denoted by \( F(w) \).

\(^6\)In a previous working paper version (Holzner and Larch, 2011) we also consider a more general convex
vacancy creation cost function \((c/\alpha) v^\alpha\) with \( \alpha > 1 \).
### 3.2 Product Market and New Product Ideas Market

Similar to Ludema (2002) consumers’ utility function is given by,

\[
U = \frac{1}{\rho} \left[ \int_{\omega \in \Omega} q(\omega)^\rho \, d\omega \right] + q_0,
\]

where \(0 < \rho < 1\) and \(\Omega\) equals the mass of available varieties and \(q_0\) is an outside good serving as numéraire.\(^7\) Each variety \(\omega\) is produced by a single firm in a monopolistic competitive market. The mass of producers is equal to the number of active firms \(M\) in the market.

Assume that there are \(n+1\) identical countries that differ only in the variety \(\Omega\) of goods they produce. Given that consumers love variety, they are interested in foreign varieties. Consumers’ utility maximization leads to the following demand for variety \(\omega\),

\[
q(\omega) = p(\omega)^{-1/\tau}.
\]

Due to the symmetry of countries, producers face the same demand curve in each export market as they face in the domestic market. Serving an export market involves some proportional shipping costs \(\tau\) per unit exported. Thus, the price of an export good at the factory gate is given by \(p_x(\omega)/\tau = p_d(\omega)\), where \(p_d(\omega)\) denotes the price in the domestic market. A firm that decides to serve \(j \leq n\) export markets will choose the output sold in each of the \(j\) export markets \(q_x(\omega)\) and the domestic market \(q_d(\omega)\) such that profits are maximized (see derivation in Appendix A),

\[
q_d(\omega) = \frac{1}{1 + j\tau^{\rho}/(\rho-1)} q(\omega), \quad \text{and}
\]

\[
q_x(\omega) = \frac{\tau^{\rho}/(\rho-1)}{1 + j\tau^{\rho}/(\rho-1)} q(\omega).
\]

Producers use only labor \(l(\omega)\) in production. As in Melitz (2003), firms differ in labor productivity such that the output of a firm that produces good \(\omega\) is given by \(q(\omega) = \varphi(\omega) l(\omega)\), where \(\varphi(\omega)\) denotes the labor productivity of goods producer \(\omega\). As is standard in the literature, we use \(\varphi\) to index goods producers.\(^8\) Due to the monopsony power in the labor market of goods producers in the Burdett-Mortensen model, the size \(l(w)\) of the labor force employed by a firm depends on its wage \(w\).

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\(^7\)The numéraire good \(q_0\) in the utility function in equation (2) absorbs all changes in aggregate demand and is assumed to be costlessly tradable, balancing trade between countries.

\(^8\)We can think of \(\varphi\) as labor productivity or as quality of a product idea. With the given form of product differentiation, these two interpretations are isomorphic (see Melitz, 2003, page 1699 and footnote 7). We use quality and productivity interchangeably, since goods firms only care about profit per unit of labor.
Goods producers are risk neutral and live forever\(^9\). Each firm is concerned about its steady-state profit flow. We assume that a goods producers’ demand totally breaks down at the Poisson rate \(\delta\), reflecting the end of a specific variety’s product cycle\(^10\). The rate \(\delta\) acts as discount rate for firms. Thus, a firm that pays a wage \(w\) and serves \(j\) export markets has a steady-state profit flow equal to the per period revenues from serving the domestic and \(j\) export markets minus the wage bill, production fixed costs \(f\), and exporting fixed costs \(f_x\) for each foreign market, i.e.,

\[
\delta \Pi (w, j|\varphi) = \left[ 1 + j \tau \rho \frac{\rho}{\tau - 1} \right] (1 - \rho) \left[ \varphi l (w) \right] - w l (w) - f - j f_x.
\]  

(6)

A firm with productivity \(\varphi\) maximizes profits by deciding on the wage offer \(w(\varphi)\) and on the number \(j(\varphi)\) of export destinations. Denote by \(w(\varphi)\) and \(j(\varphi)\) the wage offer and the number of export destinations that maximize equation (6) for a firm with productivity \(\varphi\), respectively. Furthermore, denote by \(\Pi (\varphi)\) the maximum discounted profits that a firm with a product idea \(\varphi\) can make, i.e.,

\[
\Pi (\varphi) = \max_{w,j} \Pi (w, j|\varphi).
\]

If demand for a specific variety breaks down at rate \(\delta\), a firm will acquire a new product idea in the market for product ideas\(^11\). Product ideas are sold in a perfectly competitive market. Existing goods producers that compete for new product ideas differ in their stock of labor. In the Burdett-Mortensen model, the stock of workers \(l(w(\varphi))\) that a firm employs depends on the wage \(w(\varphi)\) that the firm committed itself to pay to all its workers for their entire employment spell. A firm with labor force \(l(w(\varphi))\) that pays the wage \(w(\varphi)\) is thus willing to bid up to \(\Pi (\varphi)\) for a product idea \(\varphi\). Since there are always several firms with a labor force arbitrarily close to \(l(w(\varphi))\) the price for the product idea is bid up to \(\Pi (\varphi)\).

Inventors of new product ideas have to invest in research and development at cost \(f_e\) before they will know the quality of the idea. As it is common in the literature we assume

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\(^9\)This standard assumption in the Burdett-Mortensen model allows us to refrain from considering the effects of firm growth.

\(^10\)Note that we do not aim at modeling the life-cycle of the product itself. See Vernon (1966) and Klepper (1996) for the idea of product life cycles.

\(^11\)We separate inventors and firms, because otherwise we would have to model firm growth. When firms also invest, new firms that enter the market would only post \(v\) vacancies due to convex vacancy creation costs. Hence, they would grow steadily up to the optimal steady state level. This would change firms’ position in the wage offer distribution over time and violate the rank preserving assumption that is needed to solve a dynamic version of the Burdett-Mortensen model (see Moscarini and Postel-Vinay, 2013).
that product idea qualities $\varphi$ are drawn from a Pareto-distribution denoted by $\Gamma (\varphi)$. Since existing firms are only willing to buy profitable product ideas, only products with quality $\varphi \in [\varphi^*, \infty)$ will be available in the market, where $\varphi^*$ is defined as the cutoff productivity, i.e.,

$$\Pi (\varphi^*) = 0.$$  \hspace{1cm} (7)

The distribution of qualities sold in the market is therefore given by

$$\tilde{\Gamma} (\varphi) \equiv \left[ \Gamma (\varphi) - \Gamma (\varphi^*) \right] / \left[ 1 - \Gamma (\varphi^*) \right] = 1 - (\varphi^*/\varphi)^\gamma \text{ with } \gamma > 1. \quad (\gamma)$$

Since all product ideas (except $\varphi^*$) make positive profits, the expected discounted profits before knowing $\varphi$ is given by,

$$\Pi_e = \left[ 1 - \Gamma (\varphi^*) \right] \overline{\Pi} = \left[ 1 - \Gamma (\varphi^*) \right] \int_{\varphi^*}^{\infty} \frac{\Pi (\varphi)}{1 - \tilde{\Gamma} (\varphi^*)} d\Gamma (\varphi) > 0,$$

where $1 - \Gamma (\varphi^*)$ equals the probability that an inventor will draw a $\varphi$ high enough to be profitable. $\overline{\Pi}$ equals the average discounted profits of all product ideas available in the market.

Free entry of inventors ensures that new product ideas enter the market until the expected discounted profits before entering the market equal the fixed investment costs $f_e$, i.e.,

$$\left[ 1 - \Gamma (\varphi^*) \right] \overline{\Pi} = \int_{\varphi^*}^{\infty} \Pi (\varphi) d\Gamma (\varphi) = f_e. \quad (8)$$

The zero cutoff profit condition in equation (7) and the free entry condition in equation (8) determine the number of active firms $M$ in the product market\(^{12}\) and the productivity $\varphi^*$ of the firm with the lowest productivity in the economy.

### 3.3 Aggregation and Steady-State Conditions

Aggregate profits are used to finance new product ideas and thus the initial research and development costs of inventors, i.e.,

$$M \delta \overline{\Pi} = f_e I_e. \quad (9)$$

$I_e$ is the total mass of inventors who attempt entry and pay the fixed investment costs $f_e$ each period. A large unbounded set of potential new product ideas ensures an unlimited supply of potential entrants into the market for new product ideas. Steady state requires that the flow into the pool of new product ideas is equal to the inflow of existing firms that want to buy new product ideas, i.e.,

$$\left[ 1 - \Gamma (\varphi^*) \right] I_e = \delta M. \quad (10)$$

\(^{12}\)The number of active firms $M$ enter the zero cutoff profit condition in equation (7) and the free entry condition in equation (8) through the labor input $l (w (\varphi))$ given in equation (15).
It is straightforward to show that the steady-state conditions of equations (9) and (10) hold if the free entry condition in equation (8) holds.

In steady-state in- and outflows to employment offset each other such that the distribution of employment over firms and the unemployment rate are stationary. Equating the flows in and out of unemployment gives the steady-state measure of the unemployed, i.e.,

\[ u = \frac{\kappa + \phi}{\kappa + \phi + \lambda}, \]

where \( \kappa \) denotes the quitting rate into unemployment, \( \phi \) the rate at which workers exit the labor market, and \( \lambda \) the rate at which unemployed workers find a job. Equating the inflow and outflow of workers earning less than \( w \) gives the steady-state wage earnings distribution \( G(w) \), i.e.,

\[
\lambda F'(w-u) u = G(w)(1-u) \left[ \kappa + \phi + \lambda \left[ 1 - F(w) \right] \right]
\]

\[ \Rightarrow \quad G(w) = \frac{(\kappa + \phi) F(w)}{\kappa + \phi + \lambda} \left[ 1 - F(w) \right], \]

where the wage offer distribution across firms \( F(w) \) can contain mass points. If a mass point exists, then \( \vartheta(w) \) denotes the mass of firms offering a wage \( w \) and \( F'(w) \) (and similarly \( G'(w) \)) defines the limit \( \lim_{w' \to w} F(w) \), with \( F(w) = F(w) - \vartheta(w) \).

The steady-state size of a firm \( l(w) \) is determined by the hiring and quitting rates at a firm that pays wage \( w \), i.e.,

\[ l(w) = \frac{\eta(M) \bar{v} [u + (1-u) G(w^-)]}{\kappa + \phi + \lambda [1 - F(w)]}. \]

The number of recruited workers depends on the contact rate \( \eta(M) \), the number of vacancies \( \bar{v} \), and the probability that the contacted workers are willing to work for the wage \( w \). If the wage \( w \) exceeds the unemployment benefit \( z \), then all unemployed workers \( u \) and all workers employed at a lower wage will accept it, i.e., \( (1-u) G(w^-) \). The rate at which workers quit for a better paying job is given by \( \lambda [1 - F(w)] \). Substituting \( \lambda \) using the aggregate matching condition of equation (1), \( u \) using the steady-state measure of unemployed workers in equation (11), and \( G(w^-) \) using equation (13) allows us to write the steady-state labor force of a firm that pays a wage \( w \) as,

\[ l(w) = \frac{\lambda (\kappa + \phi)}{[\kappa + \phi + \lambda [1 - F(w^-)]]} \left[ \kappa + \phi + \lambda [1 - F(w)] \right] M'. \]

As in Burdett and Mortensen (1998), equation (15) implies that the size of a firm’s labor force \( l(w) \) is increasing in the wage \( w \), since firms with a high wage attract more workers from
low-paid jobs and lose fewer workers to high-paid jobs. Equation 15 also shows that a higher number of active firms $M$ results in additional competition between firms and decreases the size of each firm’s labor force.

4 Equilibrium of the Model

4.1 Equilibrium Definition

The set \{ $w(\varphi) , j(\varphi) , \Pi (\varphi) , F (w(\varphi)) , G(w(\varphi)) , \varphi^* , u , M$ \} defines a steady-state equilibrium. Thus, in equilibrium firms maximize the steady-state profit flow of equation 6 by choosing a wage offer $w(\varphi)$ and a number of export destinations $j(\varphi)$, given the wage offer distribution $F (w(\varphi))$, the number of active firms $M$ in the economy, the productivity distribution $\Gamma (\varphi)$, the cutoff productivity $\varphi^*$, and the optimal search strategy of workers. Thus, the equilibrium optimality condition requires,

\[
\delta \Pi (\varphi) = \left[ 1 + j(\varphi) \tau^{\rho - 1} \right] (1 - \rho) \left[ \varphi l (w(\varphi)) \right]^\rho - w(\varphi) l (w(\varphi)) - f - j(\varphi) f_x,
\]

\[
\delta \Pi (\varphi) \geq \left[ 1 + j(\varphi') \tau^{\rho - 1} \right] (1 - \rho) \left[ \varphi l (w(\varphi')) \right]^\rho - w(\varphi') l (w(\varphi')) - f - j(\varphi') f_x.
\]

Furthermore, in the steady-state equilibrium, the unemployment rate $u$ and the wage earnings distribution $G(w(\varphi))$ in equations 11 and 13 have to be consistent with the wage offer distribution and steady-state turnover of workers.

Inventors enter the market for new product ideas if the quality of their idea exceeds the cutoff productivity, i.e., $\varphi \geq \varphi^*$, where $\varphi^*$ is implicitly defined by equation 7. The number of active firms $M$ in the product market has to be such that the average profits of active firms are sufficient to finance the research and development costs $f_e$ of potential inventors, that is, such that the inflow of new product ideas equals the number of products whose demand breaks down. The respective steady-state conditions in equations 9 and 10, or, equivalently, in the free entry condition in equation 8 must be satisfied in equilibrium.

4.2 Firms’ Wage Offer and Export Decision

Each firm chooses a wage $w(\varphi)$ and the number of export destinations $j(\varphi)$ that maximizes its steady-state profit flow. In the presence of on-the-job search and convex vacancy creation costs, more productive firms pay higher wages. They are therefore able to poach workers from less productive firms, which is why exporting firms are larger than non-exporting firms.
The following Proposition shows not only that the posted wage \(w(\varphi)\) weakly increases with productivity \(\varphi\) as in Mortensen (1990) but also the number of export destinations \(j(\varphi)\).

**Proposition 1 (Wage offer and export decision)** Take two firms with productivity \(\varphi\) and \(\varphi' \in [\varphi^*, \infty)\) and assume \(\varphi > \varphi'\), then \(w(\varphi) \geq w(\varphi')\) and \(j(\varphi) \geq j(\varphi')\).

**Proof.** See Appendix B.

We will focus on an equilibrium where all firms are willing to post \(\overline{v}\) vacancies and where wages strictly increase with productivity. If unemployment benefits \(z\) and hence reservation wages are high enough, marginal profits of the least productive firm with \(\overline{v}\) vacancies could be negative. In this case, the least productive firm will want to lower the number of vacancies \(\overline{v}\) in order to increase its marginal profits. In Appendix C we show that Assumption 1 is sufficient (although not necessary) to ensure that marginal profits of the least productive firm with \(\overline{v}\) vacancies are positive and that wages are therefore strictly increasing with productivity over the whole support of the wage offer distribution.

**Assumption 1:** We assume \(z \leq \overline{z}\), where \(\overline{z}\) is defined by,

\[
(1 - \rho) \overline{z} + \frac{\kappa + \phi + \lambda}{2\lambda\gamma} (\overline{z} + f)^{1/\rho} = \rho f.
\]

Given Assumption 1 wages \(w(\varphi)\) strictly increase with \(\varphi\) like in Mortensen (1990). Thus, the position of a firm in the wage offer distribution \(F(w(\varphi))\) is equivalent to its position in the productivity distribution of active firms, i.e.,

\[
F(w(\varphi)) = 1 - \left(\frac{\varphi^*}{\varphi}\right)^\gamma \text{ for all } \varphi \in [\varphi^*, \infty). \tag{16}
\]

Since a firm’s position in the wage offer distribution determines a firm’s labor input, equation (16) implies that a firm’s production is constrained by its position in the productivity distribution of active firms. It therefore cannot freely adjust output to changes in demand and will thus adjust its price.

Proposition 1 also implies that more productive firms choose to export to (weakly) more foreign countries. We can therefore define the export cutoff productivities \(\varphi^*_j\) for \(j \in \{1, 2, \ldots, n\}\) export destinations. Firms with \(\varphi \geq \varphi^*_j\) find it optimal to export to \(j\) or more countries while firms with \(\varphi < \varphi^*_j\) will serve fewer than \(j\) foreign markets and the

\[\text{The respective equilibrium is covered by the general case of endogenous vacancy creation outlined in our extension described in Holzner and Larch (2011).}\]
domestic market (or only the domestic market). The firm with the export cutoff productivity \( \varphi^j \) is indifferent between serving \( j \) export markets and the domestic market or serving \( j - 1 \) export markets and the domestic market, i.e., \( \delta \Pi \left( w, j | \varphi^j \right) = \delta \Pi \left( w, j - 1 | \varphi^j \right) \). Using equation (6), the export cutoff productivity can be written as follows,

\[
\varphi^j = \frac{f_x}{\left[ 1 + j \tau \rho/(\rho - 1) \right]^{(1-\rho)} - \left[ 1 + (j - 1) \tau \rho/(\rho - 1) \right]^{(1-\rho)}}.
\]

Equation (17) implies, in accordance with Proposition 1, that the number of export markets \( j \leq n \) served by a firm is increasing in its productivity.

Given the export destination decision of firms as defined by the export cutoff productivities \( \{ \varphi^j \}_{j=1}^n \), we can derive the wage paid by a firm with productivity \( \varphi \). Since a firm has to pay at least the reservation wage in order to attract workers, the least productive firm \( \varphi^* \) will offer a wage equal to unemployment benefits \( z \). In Appendix D we derive the optimal wage \( w(\varphi) \) posted by a firm with productivity \( \varphi > \varphi^* \), i.e.,

\[
w(\varphi) = \frac{1}{l(w(\varphi))} \left[ 1 + j (\varphi) \tau \rho/(\rho - 1) \right]^{(1-\rho)} \left[ \varphi l(\varphi) \right]^{\rho} - f - j f_x \]

\[
- \frac{1}{l(w(\varphi))} \sum_{i=1}^{j(\varphi)+1} \left[ 1 + (i - 1) \tau \rho/(\rho - 1) \right]^{(1-\rho)} \int_{\varphi^{i-1}}^{\varphi^i} \frac{\rho}{\varphi} \left[ \varphi l(\varphi) \right]^{\rho} d\varphi,
\]

where \( \varphi^0 = \varphi^*, \varphi^{j+1} = \varphi \) and \( j \leq n \).

Note that profit maximization ensures that the wage function does not jump upward at \( \varphi^j \), that is, the support of the wage distribution is connected. To see this, suppose the opposite, that is, that the exporting firm with the lowest productivity \( \varphi^j \) would pay a wage \( w(\varphi^j) = w(\varphi) + \Delta \), where \( \Delta > 0 \) denotes the jump at \( w(\varphi) \) where productivity is given by \( \varphi = \varphi^j - \varepsilon \) for any small \( \varepsilon > 0 \). The wage jump does not increase the number of workers of firm \( \varphi^j \) since it has the same position in the wage distribution as before. It is therefore optimal for the firm to pay a wage that is only slightly above \( w(\varphi) \) and save the wage costs \( \Delta \) per worker. Thus, the wage function is continuous on \( [\varphi^*, \infty) \).

Multiplying equation (18) by \( l(w(\varphi)) \) reveals that total wage payments are given by revenues (the first term on the right-hand side in brackets) minus fixed production and export costs minus the profits of a firm with productivity \( \varphi \) (the second line on the right-hand side).

### 4.3 Firm Entry Decision

Free entry of potential inventors ensures that the expected discounted profits earned in the product market \( [1 - \Gamma (\varphi^*)] \Pi \) are used to finance the fixed investment costs \( f_e \) as stated in
equation (8). Substituting per period profits from equation (6) and the optimal wage from equation (18) implies the following free entry condition for product idea inventors,

$$f_e = \frac{1}{\delta} \int_{\varphi^*}^{\infty} \left[ \frac{j(\varphi)+1}{1+\rho-\rho} \right] \left[ \int_{\varphi_x^{j+1}}^{\varphi^*} \rho \frac{l(w(\varphi))}{\varphi} d\varphi \right] d\Gamma(\varphi), \quad (19)$$

where $\varphi_x^0 = \varphi^*$, $\varphi_x^{j+1} = \varphi$ and $j \leq n$. The derivation of equation (19) is given in Appendix E.

The expected discounted profits decrease with the number of active goods producers because the size of a firm’s labor force $l(w(\varphi))$ is a decreasing function of the number of active firms $M$. At the same time, the expected discounted profits of an inventor increase if the cutoff productivity decreases because the likelihood of having a productivity draw that can be sold to a goods producer increases. Using the implicit function theorem, we show in Appendix F that the free entry condition defines a decreasing relation between the zero cutoff productivity $\varphi^*$ and the number of active goods producers $M$ in the market.\(^{14}\)

![Figure 1: Number of total vacancies ($M \overline{v}$) and cutoff productivity](image)

A goods producer has to offer at least the level of unemployment benefits $z$ in order to attract any worker. Given this lower bound of the support of the wage offer distribution $F(w(\varphi))$, the zero cutoff productivity firm $\varphi^*$ employs $l(z) = l(w(\varphi^*))$ workers. Utilizing\(^{14}\)

\[^{14}\] We displayed on the x-axis the number of active firms $M$ times the fixed number of vacancies $\overline{v}$ posted by each firm in order to also illustrate, in Section 5.3, the effect of trade liberalization without convex vacancy creation costs.
the per period profits definition from equation (6) and the expression for \( l(w(\varphi^*)) \) from equation (15) implies that the zero cutoff productivity level \( \varphi^* \) is given by,

\[
\varphi^* = \frac{\lambda(\kappa + \phi)}{[\kappa + \phi + \lambda]^2 M} = \frac{z \lambda(\kappa + \phi)}{[\kappa + \phi + \lambda]^2 M} + f. \tag{20}
\]

Since the zero cutoff productivity firm pays the reservation wage \( z \), it will attract only unemployed workers and lose its current workers to all other firms that pay higher wages. Consequently, a higher number of active firms \( M \) increases the number of quits at the zero cutoff productivity firm and therefore reduces its steady-state labor input. This decreases the firm’s net revenue. The firm will subsequently no longer be able to cover the wage payments or the fixed costs \( f \). Thus, only more profitable firms will survive, which increases the zero cutoff productivity. Using the implicit function theorem and Assumption 1 we show in Appendix F that the zero cutoff profit condition defines an increasing relation between the zero cutoff productivity \( \varphi^* \) and the number of active goods producers \( M \) in the market. Thus, the free entry condition and the zero profit condition determine a unique equilibrium, as shown in Figure 1.

5 The Importance of On-The-Job Search and Convex Vacancy Creation Costs

In this section we first describe the additional frameworks without on-the-job search and with linear vacancy creation costs that allow us to highlight the role of on-the-job-search and convex vacancy creation costs. We then characterize the equilibria of our different frameworks, followed by a comparative static analysis of the effects of trade liberalization on trade patterns, wages, and wage inequality.

5.1 Additional Cases

To disentangle the influence of convex vacancy creation costs and on-the-job search on our results, we consider in addition to our current framework all other combinations with convex versus linear vacancy creation costs and with and without on-the-job search.

**Without On-The-Job Search and Convex Vacancy Creation Costs**

Ruling out on-the-job search implies that all firms are restricted to hiring only from the pool of unemployed workers. Abolishing on-the-job search therefore deprives more productive firms
of the possibility to poach workers from less productive firms by paying higher wages. Since unemployed workers are willing to work for the reservation wage \( z \), firms have no incentive to increase wages above the reservation wage \( z \), and all firms offer a wage equal to \( z \). If all firms offer the same wage, the wage distribution degenerates to a mass point at \( z \), i.e., \( \vartheta(z) = 1 \). This, together with the assumption of convex vacancy creation costs, which allows firms to offer only a maximum of \( \varpi \) vacancies, implies that all firms employ the same number of workers,

\[
 l(z) = \frac{\lambda}{(\kappa + \phi + \lambda) M}.
\]  

Equation (21) can be obtained by using equation (15) and noting that \( \vartheta(z) = 1 \) implies \( F(z) = 1 \) and \( F(z^-) = 0 \).

**With On-The-Job Search and Linear Vacancy Creation Costs**

The previous analysis was based on the assumption that firms cannot expand their labor input by opening new vacancies. In this subsection we allow firms to influence their contact rate by posting vacancies, as in Mortensen (2003), given linear vacancy creation costs \( cv \). The contact rate of a firm with productivity \( \phi \) depends on the number of vacancies \( v(\phi) \) it posts. Let \( \bar{v} \) denote the average number of vacancies posted by all \( M \) active firms in the economy. Firms will choose the vacancies \( v(\phi) \) such that profits are maximized. The average number of vacancies must satisfy,

\[
\bar{v} = \int_{\phi^*}^{\infty} \frac{v(\phi)}{1 - \Gamma(\phi^*)} d\Gamma(\phi).
\]

A firm with productivity \( \phi \) chooses its wage \( w \), its number of vacancies \( v \), and its export destinations \( j \) such that per period profits are maximized, i.e.,

\[
\delta\Pi(\phi) = \max_{w,v,j} \left[ 1 + j + \frac{\rho}{\rho - 1} \left[ \varphi l(w,v) \right] \left[ w(l(w,v)) - cv - f - jf \right] \right],
\]

\[
\text{s.t. } l(w,v) = \frac{\lambda(\kappa + \phi) v}{\kappa + \phi + \lambda [1 - \Gamma(w^-)] [\kappa + \phi + \lambda [1 - F(w)]] \tilde{M}v}.
\]

The number of employees \( l(w,v) \) working for a firm with productivity \( \phi \) increases proportionally with the number of vacancies \( v \), as in Mortensen (2003), and with the wage, as in Burdett and Mortensen (1998). Thus, firms can increase their labor input by increasing the wage and by opening more vacancies.

**Without On-The-Job Search and Linear Vacancy Creation Costs**
If we allow for vacancy creation, but rule out on-the-job search, again all firms will find it optimal to pay the reservation wage $z$. Firms will hire only unemployed workers, which implies that the labor force at a firm that posts $v$ vacancies is given by,

$$l(z,v) = \lambda v \left(\zeta + \phi + \lambda \right) M \tilde{v}. \quad (24)$$

This equation can also be obtained by substituting $\vartheta(z) = 1$, which implies $F(z) = 1$ and $F(z^-) = 0$, into equation (23).

In all cases the free entry condition and the zero profit condition determine a unique equilibrium as shown in Figure 1. In Appendix F we cover both frameworks with convex vacancy creation costs and in Appendix G we derive the free entry and the zero cutoff condition for firms with linear vacancy creation costs and prove the uniqueness of the equilibrium.

### 5.2 Equilibrium Characterization

In the presence of convex vacancy creation costs and on-the-job search, more productive firms pay higher wages. They are therefore able to poach workers from less productive firms, which implies that the labor input of a firm increases with productivity. If, instead, firms can only hire unemployed workers, they will pay the reservation wage $z$.

With linear vacancy creation costs and on-the-job search, wages also increase with productivity. The reason firms pay different wages is the same as in the simple Burdett-Mortensen model. To see this, suppose the contrary holds, that is, all firms paid the same wage. Then each firm would have an incentive to deviate and offer a slightly higher wage, since it will then also be able to recruit workers employed at other firms at no extra cost, that is, without paying extra vacancy creation costs. The respective wage equation can be obtained by rearranging the optimality conditions for wages and vacancies (as shown in Appendix H),

$$w(\varphi) = c \left[ \frac{v(\varphi^*)}{l(z,v(\varphi^*)))} - \frac{v(\varphi)}{l(w(\varphi),v(\varphi)))} \right] + z. \quad (25)$$

Thus, in a framework with on-the-job search and linear vacancy creation costs labor input also increases with productivity. Proposition 2 states the consequence of the different wage pattern for the exporter wage premium.

**Proposition 2 (Exporter wage premium)** Among the considered frameworks, an exporter wage premium only exists in frameworks with on-the-job search.
Proof. The Proposition directly follows from the fact that wages are increasing in productivity in all frameworks with on-the-job search, and degenerate at the reservation wage in all frameworks without on-the-job search.

We therefore need on-the-job search in order to generate a realistic wage distribution that is able to capture the stylized fact that workers in exporting firms are paid higher wages compared to similar workers at non-exporting firms.

Consider now the trade patterns that emerge under the different frameworks when a country opens up to trade. Proposition 1, together with the export cutoff equation (17), implies that in a framework with convex vacancy creation costs, firms with very low productivity serve only the domestic market, firms with higher productivity export, and more productive firms export to more countries. The assumptions of convex vacancy creation costs and export fixed costs \( f_x \) are the driving forces behind this result. With convex vacancy creation costs and fixed costs to enter an additional export market, firms will find it optimal to serve only a limited number of countries and charge higher prices in them such that overall revenues are maximized. Allowing, or not, for on-the-job search does not change the trade pattern qualitatively, only quantitatively. Formally, this can be seen by the fact that abolishing on-the-job search implies that the left-hand side of equation (17) is still increasing in the export cutoff productivity \( \phi^j_x \) even if \( \left( \phi^j_x l(z) \right)^{\rho} \) replaces \( \left( \phi^j_x l \left( w \left( \phi^j_x \right) \right) \right)^{\rho} \).

If firms face linear vacancy creation costs, they will adjust their labor input such that marginal revenues from employing a worker are equal to the costs of vacancy creation, i.e.,

\[
\rho \left[ 1 + j \left( \phi \right) \tau^{\rho \rho - 1} \right]^{(1-\rho)} \phi^{\rho l \left( w \left( \phi \right), v \left( \phi \right) \right)^{\rho - 1}} - w \left( \phi \right) \frac{l \left( w \left( \phi \right), v \left( \phi \right) \right)}{v \left( \phi \right)} = c. \tag{26}
\]

Substituting wages using equation (25) implies that firms increase their number of vacancies up to the point where marginal revenues are equalized across firms with different productivities, i.e.,

\[
\rho \left[ 1 + j \left( \phi \right) \tau^{\rho \rho - 1} \right]^{(1-\rho)} \phi^{\rho l \left( w \left( \phi \right), v \left( \phi \right) \right)^{\rho - 1}} = \frac{z + c l \left( w \left( \phi^* \right), v \left( \phi^* \right) \right)}{l \left( z, v \left( \phi^* \right) \right)}. \tag{27}
\]

The reason for the constant marginal revenue condition is that with linear vacancy creation costs firms find it optimal to increase their labor input until marginal revenues are equal to the marginal costs of creating a vacancy. This constant marginal revenue condition allows us to determine the number of workers \( l \left( w \left( \phi \right), v \left( \phi \right) \right) \) employed by a firm with productivity \( \phi \), i.e.,

\[
l \left( w \left( \phi \right), v \left( \phi \right) \right) = \left[ 1 + j \left( \phi \right) \tau^{\rho \rho - 1} \right] \left[ \frac{z}{\rho} + \frac{c l \left( w \left( \phi^* \right), v \left( \phi^* \right) \right)}{\rho l \left( z, v \left( \phi^* \right) \right)} \right]^{\frac{1}{1-\rho}} \phi^{\frac{1}{\rho}} \phi^{1-\rho} \tag{28},
\]
where \( v(\phi^*) / l(z, v(\phi^*)) \) can be substituted by noting that equation (23) implies
\[
[ v(\phi^*) \lambda (\kappa + \phi) ] / \left[ \tilde{M} \hat{v} (\kappa + \phi + \lambda)^2 \right].
\]
To obtain the respective labor input for a framework without on-the-job search and with linear vacancy creation costs one just needs to substitute
\[
v(\phi^*) / l(z, v(\phi^*))
\]
using equation (24).

Taking the profit equation (22), substituting wage and vacancy creation costs using the optimality condition for vacancy posting in equation (26), and substituting labor input
\[
l(w(\phi), v(\phi))
\]
using equation (28) implies that profits are linear in the number of export destinations, i.e.,
\[
\delta \Pi (\phi, j) = (1 - \rho) \left[ 1 + j \tau \rho^{\phi} \right] \left[ z + \frac{c}{\rho} l(z, v(\phi^*)) \right] \frac{1 - \rho^{\phi}}{\tau^{1 - \rho} - f - j f_x},
\]
where \( v(\phi^*) / l(z, v(\phi^*)) \) can again be substituted by using equation (23). Equation (29)
holds also for the case without on-the-job search, where \( v(\phi^*) / l(z, v(\phi^*)) \) can be substituted
using equation (24).

We can now use the profit equation (29) to obtain the export cutoff, i.e.,
\[
\delta \Pi (\phi_x, j) = \delta \Pi (\phi_x, j - 1).
\]
The export decision is independent of \( j \), the number of export destinations, i.e.,
\[
\phi_x = \frac{\tau}{\rho} \left[ \frac{f_x}{(1 - \rho)} \right]^{\frac{1 - \rho}{\tau^{1 - \rho}}} \left[ z + \frac{c}{\rho} l(z, v(\phi^*)) \right].
\]
Since the cost to adjust labor input to foreign demand is linear in output, firms that find it profitable to serve only one foreign market also find it profitable to serve all foreign markets. Thus, all exporting firms serve all \( n \) export markets. Proposition 3 summarizes the trade patterns in the different frameworks.

**Proposition 3 (Trade pattern)** The number of export destinations \( j \) is weakly increasing in productivity in all frameworks with convex vacancy creation costs. In all frameworks with linear vacancy creation costs, all exporting firms serve all \( n \) foreign countries.

**Proof.** See Appendix for the proof that the number of export destinations \( j \) increases with productivity in all frameworks with convex vacancy creation costs. The result for linear vacancy creation costs follows directly from equation (30). ■

Proposition 3 shows the importance of convex vacancy creation costs for obtaining a realistic trade pattern, where the number of export destinations increases with firm productivity. If we assume a Pareto distribution for firm productivity, as it is common in the trade literature, then convex vacancy creation costs explain why most exporting firms sell to only
one foreign market and why the number of firms that sell to multiple markets declines with the number of destinations, as documented by Arkolakis (2010) and Eaton, Kortum, and Kramarz (2011).

Eaton, Kortum, and Kramarz (2011) have also shown that the export strategies of observationally equivalent firms vary greatly across countries with similar characteristics. This can also be explained by convex vacancy creation costs because exporting firms that face convex vacancy creation costs do not find it optimal to adjust their production so as to serve all foreign markets. Thus, even if two export markets are similar and firms indifferent between them, firms might export to just one country and not the other, because they find it too costly to increase production to serve both countries.

5.3 Comparative Statics

Let us now investigate the effects of trade liberalization in the form of lower shipping costs $\tau$ and lower exporting fixed costs $f_x$.\footnote{In Holzner and Larch (2011), a previous working paper version of this paper, we show in a calibration exercise that the same results hold for an exponential convex vacancy creation cost function if we switch from autarky to an open economy.}

Lower shipping costs $\tau$ and lower exporting fixed costs $f_x$ directly increase exporting firms’ profits. The associated increase in average profits rotates the free entry curve outward as shown in Figure 2. This is true for all frameworks as shown in Appendix J.

The zero cutoff profit condition in equation (20) remains unchanged if trade is liberalized, since the firm with the lowest productivity is not affected as it pays the reservation wage $z$ and serves only the domestic market. The outward rotation of the free entry condition shows that higher average profits caused by trade liberalization trigger entry. In a framework with convex vacancy creation costs where the number of vacancies per firm is fixed at $\bar{v}$, entry occurs at the extensive margin, that is, the number of active firms $M$ increases. In a framework with linear vacancy creation costs, entry can occur along the extensive and intensive margin. The outward rotation of the free entry condition alone only implies a higher number of vacancies $\tilde{M}$ in the economy. Given the increased number of vacancies in the economy in either framework, potential entrants realize that more vacancies are competing for the same number of workers, which implies that only more productive firms can survive and that the cutoff productivity $\varphi^*$ has to increase.

To investigate the effect of trade liberalization on the trade pattern consider the ratio
Figure 2: Number of total vacancies \((M\bar{v})\) and cutoff productivity if trade is liberalized

of the export cutoff productivities to the zero profit cutoff productivity \(\phi^*\) for the cases of convex and linear vacancy creation costs respectively, i.e.,

\[
\left[ \frac{\phi^* l(z)}{\phi_x^* l\left( w\left( \phi_x^* \right) \right)} \right]^\rho = \frac{f}{f_x} \left[ \left[ 1 + j \tau \frac{\rho}{\rho-1} \right]^{(1-\rho)} - \left[ 1 + (j-1) \tau \frac{\rho}{\rho-1} \right]^{(1-\rho)} \right],
\]

(31)

\[
\frac{\phi^*}{\phi_x} = \frac{1}{\tau} \left[ \frac{f}{f_x} \right]^{\frac{1-\rho}{\rho}}.
\]

(32)

These ratios \(\phi^*/\phi_x^*\) determine the fraction of firms that export to at least \(j\) countries in the frameworks with convex vacancy creation costs, i.e., \(1 - \bar{\Gamma} \left( \phi_x^j \right)\). In the frameworks with linear vacancy creation costs, the fraction of firms that export \((1 - \bar{\Gamma} (\phi_x))\) increases with \(\phi^*/\phi_x\).

Trade liberalization therefore increases the fraction of exporters in all frameworks. If vacancy creation costs are linear, all firms that start to export will export to all \(n\) export destinations, since linear vacancy creation costs allow them to expand their labor input at constant cost to meet all foreign demand. If vacancy creation costs are convex, less productive exporters export to fewer countries than do the more productive exporters. The reason for this trade pattern is that firms with convex vacancy creation costs find it optimal to serve only a limited number of countries. The following Proposition states the effect of trade liberalization on trading patterns in the different frameworks.

**Proposition 4 (Trade liberalization: Trade pattern)** A decrease in shipping costs \(\tau\) or exporting fixed costs \(f_x\) increases the fraction of exporting firms. Furthermore
(i) if vacancy creation costs are linear, all new exporters serve all n foreign countries, but
(ii) if vacancy creation costs are convex, the fraction of domestic firms and firms that export
to \( j \in \{1, \ldots, n - 1\} \) countries decreases, while the fraction of firms that export to n countries increases.

**Proof.** See Appendix K ■

Comparing the effect of trade liberalization in all frameworks with a linear vacancy cre-
ation cost function with the effect in all frameworks with a convex vacancy creation cost function again confirms that the frameworks with convex vacancy creation costs are better able to describe how trade liberalization affects the trade pattern observed in reality. Part (ii) of Proposition 4, which characterizes the impact of trade liberalization in the frameworks with convex vacancy creation costs, predicts that trade liberalization leads to an overproportional increase in the number of firms that export to n countries. The following Corollary shows that for the assumed vacancy creation cost function the tendency that more firms export to all n countries is not strong enough to eliminate the trade pattern that is typical for convex vacancy creation costs, that is, that the number of export destinations j is weakly increasing in firm productivity.\(^{16}\)

**Corollary 5** If trade is fully liberalized, i.e., \( f_x \to f \) and \( \tau \to 1 \),
(i) all firms export to all n foreign countries if vacancy creation costs are linear, but
(ii) some firms serve only the domestic market and the number of export destinations j among exporters increases with productivity \( \varphi \) if vacancy creation costs are convex.

**Proof.** See Appendix K ■

If not all firms export then this has implications for the effect of trade liberalization on
the wage distribution, as we will show below.

Wages remain unchanged in all frameworks where firms only hire unemployed workers
(without on-the-job search), since firms find it optimal to offer unemployed workers only the
reservation wage \( z \). If firms also hire workers from other firms (frameworks with on-the-
job search), wages increase when trade is liberalized (with the exception of those workers
that earn the reservation wage). This follows from the fact that exporting firms profit from

\(^{16}\)The proof is based on the vacancy creation cost function used in this paper. In Holzner and Larch (2011)
we provide simulations which suggest that the trade pattern persists even with mild convex vacancy creation
costs.
lower shipping costs or lower exporting fixed costs. This increases exporting firms’ incentive to increase their labor input by offering higher wages in order to attract more workers. In equilibrium, less productive firms are also forced to increase their wages (if they are not forced out of the market) because the increased profit due to lower shipping or lower exporting fixed costs triggers firm entry and thereby increases firms’ competition for workers.

Proposition 6 (Trade liberalization: Wages) A decrease in shipping costs $\tau$ or exporting fixed costs $f_x$ increases all wages (except the reservation wage) in all frameworks with on-the-job search, and leaves wages unchanged in all frameworks without on-the-job search.

Proof. See Appendix L.

The fact that wages at the upper end of the wage distribution increase but the wage at the least productive firm does not (it pays a reservation wage $z$) does not necessarily imply that wage inequality increases as the relative change of lower and higher wages could still be identical. We use Lorenz dominance to measure lower inequality. In order to be able to derive analytical results for the change in wage inequality we normalize unemployment benefit to zero, i.e., $z = 0$. Lorenz dominance is consistent with lower inequality according to a wide class of inequality measures, most prominently the Gini coefficient. Take two wage distributions $G_0 (w)$ and $G_1 (w)$, then $G_0 (w)$ Lorenz dominates $G_1 (w)$ if and only if,

$$\frac{\int_0^G w (G_0') dG_0'}{\int_0^1 w (G_0') dG_0'} \geq \frac{\int_0^G w (G_1') dG_1'}{\int_0^1 w (G_1') dG_1'},$$

for all quantiles $G \in [0, 1]$ and for some $G$ with strict inequality. In the case of linear vacancy creation costs and on-the-job search, trade liberalization does not directly affect wages, but it does affect wages indirectly via the total number of vacancies $M \tilde{v}$. Since wages are linear in the total number of vacancies, as one can easily see by substituting $v (.) / l (w (.), v (.) )$ in equation (25) using equation (23), wages relative to the average wage $\int_0^1 w (G_0') dG_0'$ remain unaffected by changes in $M \tilde{v}$ caused by trade liberalization. Equation (33) therefore implies that trade liberalization affects wages proportionally, which leaves inequality unchanged.

Similarly, the indirect effect via the number of active firms $M$ drops out in our framework with on-the-job search and convex vacancy creation costs. However, in our framework wages are also directly affected by lower shipping costs or lower exporting fixed costs (via the export cutoff productivities). Since these direct effects on wages are not proportional, we obtain the result that trade liberalization only affects wage inequality in the framework with on-the-job search and convex vacancy creation costs. This is summarized in the following Proposition.
Proposition 7 (Trade liberalization: Wage inequality) For \( z = 0 \), a decrease in shipping costs \( \tau \) or exporting fixed costs \( f_x \) affects wage inequality (measured by Lorenz dominance) only in a framework with on-the-job search and convex vacancy creation costs.

Proof. See Appendix L.

With sufficiently high vacancy creation costs, which prevent the least productive firms from exporting, firms’ profits in an open economy are more dispersed than under autarky. This causes wage inequality in an open economy to be higher than under autarky. Thus, wage inequality increases if a country opens up to trade. If a country that already trades with other countries decreases trade costs, wage inequality can increase or decrease. As more and more firms export, the profitability gaps and, hence, the wage gaps between domestic and exporting firms increases wage inequality at the lower end of the wage distribution. At the same time, more and more exporting firms will start to export to all \( n \) countries due to trade liberalization, which decreases the profitability gaps and, hence, the wage gaps at the upper end of the wage distribution. It is unclear which effect dominates.

Corollary 8 For \( z = 0 \), wage inequality in an open economy (including free trade, i.e., \( f_x \rightarrow f \) and \( \tau \rightarrow 1 \)) is higher than under autarky (in a framework with on-the-job search and convex vacancy creation costs).

Proof. See Appendix L.

While Helpman, Itskhoki, and Redding (2010) find an inverse u-shape relationship between trade liberalization and wage inequality, Corollary 8 shows that such an inverse u-shape does not necessarily occur in our framework. We find that wage inequality under free trade, i.e., \( f_x \rightarrow f \) and \( \tau \rightarrow 1 \), is higher than under autarky if vacancy creation costs are (sufficiently) convex. This complements the results by Helpman, Itskhoki, and Redding (2010), who find that free trade implies the same wage inequality as is present under autarky. The result in Helpman, Itskhoki, and Redding (2010) is driven by the trade pattern that emerges in their framework. Like in our linear vacancy creation cost case, they find that all firms will export to all \( n \) foreign countries if trade is fully liberalized. Thus, all firms profit equally if trade is fully liberalized. This leaves wage inequality the same as in autarky.\(^{17}\) In our model

\(^{17}\)In general Helpman, Itskhoki, and Redding (2010) find an effect of trade liberalization on wage inequality although they assume linear vacancy creation costs. The reason for this is that in their paper worker heterogeneity and the associated screening mechanism lead to a non-proportional effect of trade liberalization on wages.
with sufficiently high convex vacancy creation costs and on-the-job search, the pattern of
trade in which the number of export destinations increases with productivity remains intact
even if trade is fully liberalized. Thus, under free trade, domestic and exporting firms can
be affected differently by trade openness implying that wage inequality under free trade is
higher than under autarky.

6 Conclusions

We introduce convex vacancy creation costs and on-the-job search into a Melitz (2003)-type
new trade model with monopolistic competition and heterogeneous firms resulting in a simple
and analytically tractable model that is in line with many empirical facts about both the trade
and labor market side. To obtain realistic trade patterns, we need convex vacancy creation
costs. To obtain a realistic wage distribution, we need on-the-job search. To get an effect of
trade liberalization on residual wage inequality, we need both.

The result that trade liberalization affects residual wage inequality only in a model with
convex vacancy creation costs and on-the-job search follows from the fact that with linear
vacancy creation costs, firms post vacancies until marginal profits are equal to the constant
cost of vacancy creation. This implies that all firms – irrespective of their export strategy
– have the same marginal profit in equilibrium. Thus, wages paid in equilibrium will not
depend on a firms’ export strategy, but only on aggregate labor market tightness. Trade
liberalization, which increases vacancy creation, therefore increases all wages proportionally,
but leaves wage inequality unchanged, since all commonly used wage inequality measures
(e.g., Gini coefficient) are invariant to proportional changes. With convex vacancy creation
costs, marginal profits and thus wages depend on the number of destinations to which a firm
exports. Residual wage inequality therefore increases with trade liberalization.

If vacancy creation costs are sufficiently convex, then even if trade is fully liberalized,
not all firms will export and not all exporters will serve all markets. With linear vacancy
creation costs, free trade implies that all active firms in the economy serve all markets. This
difference in trade patterns has implications for the relationship between trade liberalization
and wage inequality. With linear vacancy creation costs, all firms export and increase their
wages proportionally. Thus, wage inequality is the same as in autarky. If vacancy creation
costs are sufficiently convex, not all firms export even if trade is fully liberalized. Then wage
inequality is higher in an open economy than under autarky.
Our theoretical model suggests that studies employing structural estimation to evaluate the extent to which trade liberalization explains the documented rise in wage inequality should not only capture firm level employment and wage patterns but also the richness of firms’ exporting behavior.

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**Appendix**

A. **Quantities Sold in the Domestic and Each Export Market**

An exporting firm that decided to serve \( j \) foreign countries maximizes its profits by equalizing marginal revenues across markets. Revenues of an exporting firm are given by,

\[
R(\varphi) = p_d(\varphi) q_d(\varphi) + \frac{j}{\tau} p_x(\varphi) q_x(\varphi)
\]

\[
= p_d(\varphi) [q(\varphi) - jq_x(\varphi)] + \frac{j}{\tau} p_x(\varphi) q_x(\varphi)
\]

\[
= [q(\varphi) - jq_x(\varphi)]^\rho + j \left[ \frac{q_x(\varphi)}{\tau} \right]^\rho.
\]

Firms choose its domestic and export sells according to equalization of marginal revenues,

\[
\frac{\partial R(\varphi)}{\partial q_x(\varphi)} = 0,
\]

\[
\rho j [q(\varphi) - jq_x(\varphi)]^{\rho-1} = \rho j \frac{1}{\tau} \left[ \frac{q_x(\varphi)}{\tau} \right]^{\rho-1},
\]

\[
qu(\varphi) - jq_x(\varphi) = \frac{1}{\tau^{\rho/(\rho-1)}} q_x(\varphi),
\]

\[
\tau^{\rho/(\rho-1)} q(\varphi) = \left[ 1 + j \tau^{\rho/(\rho-1)} \right] q_x(\varphi).
\]

Rearranging and using the fact that \( q_d(\varphi) = q(\varphi) - jq_x(\varphi) \) implies equations (4) and (5). The revenue of an exporting firm is given by,

\[
R(\varphi) = \left[ q(\varphi) - jq(\varphi) \right]^{\tau^{\rho/(\rho-1)}} \left[ 1 + j \tau^{\rho/(\rho-1)} \right]^{\rho} + j \left[ \frac{q(\varphi)}{\tau} - \frac{\tau^{\rho/(\rho-1)}}{1 + j \tau^{\rho/(\rho-1)}} \right]^{\rho}
\]

\[
= \left[ 1 + j \tau^{\rho/(\rho-1)} \right] \left[ \frac{q(\varphi)}{1 + j \tau^{\rho/(\rho-1)}} \right]^{\rho}
\]

\[
= \left[ 1 + j \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} q(\varphi)^{\rho}.
\]
B Proof of Proposition 1

Note first that the steady state labor input equation (15) implies \( l(w(\varphi)) > l(w(\varphi')) \) for \( w(\varphi) > w(\varphi') \). The second step is to show that the profit function \( \Pi(w, j | \varphi) \) is supermodular in \( \varphi, w \) and \( j \). Supermodularity of the profit function \( \Pi(w, j | \varphi) \) is given if for any \( \{w, j\} \) and \( \{w', j'\} \) with \( \varphi > \varphi' \),

\[
\Pi\left(\max\left[w, w'\right], \max\left[j, j'\right] | \varphi\right) + \Pi\left(\min\left[w, w'\right], \min\left[j, j'\right] | \varphi'\right) \geq \Pi\left(w, j | \varphi\right) + \Pi\left(w', j' | \varphi'\right).
\]

Supermodularity for the revenue function \( \Pi(w, j | \varphi) \) follows from the fact that the same inequality holds for revenues due to the Cobb-Douglas structure of the revenue function, i.e.,

\[
\left[1 + jT_T^{\frac{1}{\rho}}(1-\rho)\right] \left[\varphi l(w)\right]^{\rho}, \text{ and because wages, production fixed costs and export fixed costs cancel out.}
\]

In the third and last step we prove by contradiction that \( \varphi > \varphi' \) implies \( w(\varphi) \geq w(\varphi') \) and \( j(\varphi) \geq j(\varphi') \). Suppose that for any \( \{w(\varphi), j(\varphi)\} \) and \( \{w'(\varphi'), j'(\varphi')\} \), where wages and the number of export destinations are chosen optimally, i.e., \( w(\varphi) \) and \( w(\varphi') \) and \( j(\varphi) \) and \( j(\varphi') \), that one of the following statements holds,

1. \( w(\varphi) < w(\varphi') \) and \( j(\varphi) \geq j(\varphi') \),
2. \( w(\varphi) \geq w(\varphi') \) and \( j(\varphi) < j(\varphi') \),
3. \( w(\varphi) < w(\varphi') \) and \( j(\varphi) < j(\varphi') \).

The following chain of inequalities for each statement (1) to (3) follows from the optimality condition required in equilibrium (first and third inequality) and supermodularity of the profit function (second inequality), i.e.,

\[
0 < \Pi\left(w(\varphi), j(\varphi) | \varphi\right) - \Pi\left(\max\left[w(\varphi), w'(\varphi')\right], \max\left[j(\varphi), j'(\varphi')\right] | \varphi\right) \leq \Pi\left(\min\left[w(\varphi), w'(\varphi')\right], \min\left[j(\varphi), j'(\varphi')\right] | \varphi'\right) - \Pi\left(\max\left[w'(\varphi'), j'(\varphi')\right] | \varphi'\right) < 0,
\]

and gives the contradiction necessary to complete the proof.

C Derivation of Assumption 1

We first show that if we fix the number of vacancies \( v = \nu \) for all firms, wages might not increase with productivity at the lower end of the wage distribution. To see this suppose that
firms with different productivities \( \varphi > \varphi' \) pay the same wage \( \tilde{w} \). If \( \tilde{w} \) is optimal for \( \varphi' \), i.e., \( \Pi'_w (\tilde{w}, j (\varphi') | \varphi') = 0 \), then \( \varphi > \varphi' \) and \( j (\varphi) \geq j (\varphi') \) implies,

\[
\delta \frac{\partial \Pi (\tilde{w}, j (\varphi) | \varphi)}{\partial \tilde{w}} = \left[ 1 + j (\varphi) \right] \left( 1 - \rho \right) \varphi^\rho (\tilde{w})^{\rho - 1} - \frac{\partial l (\tilde{w})}{\partial \tilde{w}} - l (\tilde{w}) \geq 0.
\]

Thus, the firm with productivity \( \varphi \) will optimally increase its wage, i.e., \( w (\varphi) > \tilde{w} \). If \( \tilde{w} \) is optimal for \( \varphi' \), i.e., \( \Pi'_w (\tilde{w}, j (\varphi') | \varphi) = 0 \), then again the first order condition implies \( \Pi'_w (\tilde{w}, j (\varphi') | \varphi') < 0 \), since \( \varphi' < \varphi \) and \( j (\varphi') \leq j (\varphi) \). Thus, the firm with productivity \( \varphi' \) will choose a wage \( w (\varphi') < \tilde{w} \) as long as the wage \( \tilde{w} \) is above the reservation wage, i.e., \( \tilde{w} > z \). If \( \tilde{w} = z \) both firms will post the same wage, i.e., wages are not strictly increasing in productivity.

Let us now derive Assumption 1, which ensures that wages strictly increase with productivity if all firms post \( v = \bar{v} \) vacancies. The firm with the cutoff productivity that serves only the domestic market pays the reservation wage \( z \), i.e.,

\[
\Pi (z, 0 | \varphi^*) = 0 \implies [\varphi^* l (z)]^\rho = z l (z) + f.
\]

To derive Assumption 1 we want a condition that ensures \( \Pi'_w (z, 0 | \varphi') \geq 0 \) for all \( \varphi' \in [\varphi^*, \bar{\varphi}] \). Using the cutoff condition to substitute \( [\varphi^* l (z)]^\rho \) in the first order condition of firms paying the reservation wage gives,

\[
\Pi'_w (z, 0 | \varphi^*) \geq 0 \iff [\rho f - (1 - \rho) z l (z)] \frac{1}{l (z)} \frac{\partial l (z)}{\partial z} - l (z) \geq 0.
\]

Substituting \( \frac{\partial l (z)}{\partial z} \frac{1}{l (z)} = \frac{2 \lambda f (z)}{\varphi + \phi + \lambda} \) and rearranging implies,

\[
(1 - \rho) z l (z) + \frac{\varphi + \phi + \lambda}{2 \lambda f (z)} l (z) \leq \rho f
\]

If we substitute \( f (z) = \gamma / \varphi^* \) using the fact that \( \varphi \) is Pareto distributed according to \( F (w (\varphi)) \equiv 1 - (\varphi^* / \varphi)^\gamma \) and substituting \( \varphi^* = (z l (z) + f)^{1/\rho} / l (z) \) using the zero profit condition implies

\[
(1 - \rho) z l (z) + \frac{\varphi + \phi + \lambda}{2 \lambda \gamma} (z l (z) + f)^{1/\rho} \leq \rho f. \tag{34}
\]

Note that the number of workers per firm cannot exceed one, i.e., \( l (z) \leq 1 \), since the number of workers is normalized to one. Thus, any \( z \leq \bar{z} \) satisfies inequality (34), where \( \bar{z} \) is defined as follows,

\[
(1 - \rho) \bar{z} + \frac{\varphi + \phi + \lambda}{2 \lambda \gamma} (\bar{z} + f)^{1/\rho} = \rho f.
\]
This can be seen as follows, i.e.,

\[(1 - \rho) zl (z) + \frac{x + \phi + \lambda}{2\lambda \gamma} (zl (z) + f)^{1/\rho} \leq (1 - \rho) z + \frac{x + \phi + \lambda}{2\lambda \gamma} (z + f)^{1/\rho} \cdot \]

Thus, \( z \leq \bar{z} \) is sufficient to ensure non-negative marginal profits if all firms post \( v = \bar{v} \) vacancies, i.e., to ensure \( \Pi_w (z, 0|\varphi') \geq 0 \) for all \( \varphi' \in [\varphi^*, \bar{\varphi}] \).

**D Derivation of Wage Equation (18)**

The wage equation for exporting firms follows from the first order condition and the equilibrium condition \([16]\). The first order condition implies,

\[1 = \left[ 1 + j (\varphi) \tau^{\varphi} \right]^{(1-\rho)} \varphi^\rho l (\varphi)^{(\rho-1)} - w (\varphi) \frac{\partial l (w (\varphi))}{\partial w (\varphi)} \frac{1}{l (w (\varphi))} . \]

Using the fact that wages increase with productivity implies that we can multiply the above equation with \( \partial w (\varphi) / \partial \varphi \). This gives,

\[\frac{\partial w (\varphi)}{\partial \varphi} = \left[ 1 + j (\varphi) \tau^{\varphi} \right]^{(1-\rho)} \varphi^\rho l (\varphi)^{(\rho-1)} - w (\varphi) \frac{\partial l (\varphi)}{\partial \varphi} \frac{1}{l (\varphi)}. \quad (35)\]

Define,

\[T (\varphi) = \ln [l (\varphi)] \] and \( T' (\varphi) = \frac{\partial l (\varphi)}{\partial \varphi} T (\varphi) \).

Substitution simplifies the above differential equation to,

\[\frac{\partial w (\varphi)}{\partial \varphi} + w (\varphi) \frac{\partial l (\varphi)}{\partial \varphi} \frac{1}{l (\varphi)} = \left[ 1 + j (\varphi) \tau^{\varphi} \right]^{(1-\rho)} \varphi^\rho l (\varphi)^{(\rho-1)} \frac{\partial l (\varphi)}{\partial \varphi} \frac{1}{l (\varphi)} \]

\[\frac{\partial w (\varphi)}{\partial \varphi} + w (\varphi) T' (\varphi) = \left[ 1 + j (\varphi) \tau^{\varphi} \right]^{(1-\rho)} \varphi^\rho l (\varphi)^{(\rho-1)} T (\varphi) . \]

Any solution to this differential equation has to satisfy,

\[w (\varphi) e^{T(\varphi)} = \int_{\varphi^*}^{\varphi} \left[ 1 + j (\varphi) \tau^{\varphi} \right]^{(1-\rho)} \rho \left[ \varphi e^{T(\bar{\varphi})} \right]^\rho T' (\bar{\varphi}) d\bar{\varphi} + A, \quad (36)\]

where \( A \) is the constant of integration. Note that,

\[\frac{d \left[ \varphi e^{T(\varphi)} \right]^\rho}{d \varphi} = \rho \left[ \varphi e^{T(\varphi)} \right]^\rho T' (\varphi) + \rho \varphi^{\rho-1} \left[ e^{T(\varphi)} \right]^\rho . \]

The integral can thus be written as,

\[\int_{\varphi^*}^{\varphi} \rho \left[ \varphi e^{T(\bar{\varphi})} \right]^\rho T' (\bar{\varphi}) d\bar{\varphi} = \int_{\varphi^*}^{\varphi} \left[ \frac{d \left[ \varphi e^{T(\bar{\varphi})} \right]^\rho}{d \bar{\varphi}} - \rho \varphi^{\rho-1} \left[ e^{T(\bar{\varphi})} \right]^\rho \right] d\bar{\varphi} \]

\[= \left[ \varphi e^{T(\varphi)} \right]^\rho - \left[ \varphi e^{T(\varphi^*)} \right]^\rho - \int_{\varphi^*}^{\varphi} \rho \varphi^{\rho-1} \left[ e^{T(\bar{\varphi})} \right]^\rho d\bar{\varphi} . \]
Substituting into the wage equation (36) gives,

\[ w(\varphi)e^{T(\varphi)} = \left[ 1 + j(\varphi)\tau^{\varphi} \right]^{(1-\rho)} \left[ [\varphi e^{T(\varphi)}]^\rho - [\varphi^j e^{T(\varphi)}]^{(1-\rho)} - \int_{\varphi^j}^\varphi \frac{\rho}{\varphi} [\varphi e^{T(\varphi)}]^\rho d\varphi \right] + A, \]

where,

\[ A = w(\varphi^j) e^{T(\varphi^j)} + \left[ 1 + (j(\varphi) - 1)\tau^{\varphi} \right]^{(1-\rho)} \left[ [\varphi^j e^{T(\varphi^j)}]^{(1-\rho)} - \int_{\varphi^j}^{\varphi^j} \frac{\rho}{\varphi} [\varphi e^{T(\varphi)}]^\rho d\varphi \right] + w(\varphi^j) e^{T(\varphi^j-1)}, \]

and,

\[ w(\varphi^j) e^{T(\varphi^j)} = [\varphi^j e^{T(\varphi^j)}]^{(1-\rho)} - \int_{\varphi^*}^{\varphi^j} \frac{\rho}{\varphi} [\varphi e^{T(\varphi)}]^\rho d\varphi + ze^{T(\varphi^*)}. \]

Since \( A \) depends on the wage payments of those firms with export cutoff productivities \( \varphi^j < \varphi \) we need to rewrite the wage equation as follows,

\[ w(\varphi)e^{T(\varphi)} = \left[ 1 + j(\varphi)\tau^{\varphi} \right]^{(1-\rho)} \left[ [\varphi e^{T(\varphi)}]^\rho - [\varphi^j e^{T(\varphi^j)}]^{(1-\rho)} - \int_{\varphi^j}^\varphi \frac{\rho}{\varphi} [\varphi e^{T(\varphi)}]^\rho d\varphi \right] + \left[ 1 + (j(\varphi) - 1)\tau^{\varphi} \right]^{(1-\rho)} \left[ [\varphi^j e^{T(\varphi)}]^{(1-\rho)} - \int_{\varphi^j}^{\varphi^j} \frac{\rho}{\varphi} [\varphi e^{T(\varphi)}]^\rho d\varphi \right] + \left[ 1 + (j(\varphi) - 1)\tau^{\varphi} \right]^{(1-\rho)} \left[ [\varphi^j e^{T(\varphi^j-1)}]^{(1-\rho)} - \int_{\varphi^j}^{\varphi^j} \frac{\rho}{\varphi} [\varphi e^{T(\varphi)}]^\rho d\varphi \right] \]

or,

\[ w(\varphi)e^{T(\varphi)} = \left[ 1 + j(\varphi)\tau^{\varphi} \right]^{(1-\rho)} \left[ [\varphi e^{T(\varphi)}]^\rho - \int_{\varphi^j}^\varphi \frac{\rho}{\varphi} [\varphi e^{T(\varphi)}]^\rho d\varphi \right] - \sum_{i=1}^{j(\varphi)} \left[ 1 + i\tau^{\varphi} \right]^{(1-\rho)} - \left[ 1 + (i - 1)\tau^{\varphi} \right]^{(1-\rho)} \left[ [\varphi^j e^{T(\varphi^j)}]^{(1-\rho)} - \int_{\varphi^j}^{\varphi^j} \frac{\rho}{\varphi} [\varphi e^{T(\varphi)}]^\rho d\varphi \right] \]

\[ - \sum_{i=2}^{j(\varphi)} \left[ 1 + (i - 1)\tau^{\varphi} \right]^{(1-\rho)} \int_{\varphi^{i-1}}^{\varphi^j} \frac{\rho}{\varphi} [\varphi e^{T(\varphi)}]^\rho d\varphi \]

\[ - [\varphi^j e^{T(\varphi^j)}]^{(1-\rho)} - \int_{\varphi^j}^{\varphi^j} \frac{\rho}{\varphi} [\varphi e^{T(\varphi)}]^\rho d\varphi \]
Substituting \( e^T(\varphi) = l(w(\varphi)), \) z by using the zero cutoff profit condition (20) and \( [\varphi^j_x l(w(\varphi^x))]^\rho \) using the export cutoff condition (17) gives,

\[
\begin{align*}
  w(\varphi) l(w(\varphi)) &= \left[ 1 + j(\varphi) \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} [\varphi l(w(\varphi))]^\rho - j(\varphi) f_x \\
  &- \left[ 1 + j(\varphi) \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} \int_{\varphi_x^i}^{\varphi_x^j} \frac{\rho}{\varphi} [\varphi l(w(\varphi))]^\rho \, d\varphi \\
  &- \sum_{i=2}^{j(\varphi)} \left[ 1 + (i-1) \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} \int_{\varphi_x^{i-1}}^{\varphi_x^i} \frac{\rho}{\varphi} [\varphi l(w(\varphi))]^\rho \, d\varphi \\
  &- \int_{\varphi_x^j}^{\varphi} \frac{\rho}{\varphi} [\varphi l(w(\varphi))]^\rho \, d\varphi - f.
\end{align*}
\]

The wage equation (18) follows immediately by defining \( \xi^0 = \varphi^*, \xi^i = \varphi^i_x \) for \( i \in \{1, 2, \ldots, j(\varphi)\}, \xi^{j(\varphi)+1} = \varphi \) and \( j \leq n. \)

\section*{E Free Entry Condition}
Rearranging the wage equation (18) implies that profits of an exporting firm are given by,

\[
\delta \Pi(\varphi) = \left[ 1 + j(\varphi) \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} [\varphi l(w(\varphi))]^\rho - w(\varphi) l(w(\varphi)) - f - j(\varphi) f_x
\]

\[(39)\]

\[
= \sum_{i=1}^{j(\varphi)+1} \left[ 1 + (i-1) \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} \int_{\varphi_x^{i-1}}^{\varphi_x^i} \frac{\rho}{\varphi} [\varphi l(w(\varphi))]^\rho \, d\varphi
\]

\[(40)\]

where \( \xi^0 = \varphi^*, \xi^i = \varphi^i_x \) for \( i \in \{1, 2, \ldots, j(\varphi)\}, \xi^{j(\varphi)+1} = \varphi \) and \( j(\varphi) \leq n. \) Since free entry implies \( f_e = \Pi_e = \int_{\varphi_x^*}^{\varphi} \Pi(\varphi) \gamma(\varphi) \, d\varphi, \) integrating over all firms with productivity \( \varphi \in [\varphi^*, \infty) \) implies the free entry condition for an open economy as stated in equation (19).

\section*{F Existence Proof for the Basic Model}
Applying the implicit function theorem to the free entry condition (8) implies,

\[
\frac{d\varphi^*}{dM} = -\frac{\int_{\varphi_x^*}^{\varphi} \frac{d\Pi(\varphi)}{d\varphi} \, d\Gamma(\varphi)}{\int_{\varphi_x^*}^{\varphi} \frac{d\Pi(\varphi)}{d\varphi} \, d\Gamma(\varphi) - \Pi(\varphi^*) \frac{d\Gamma(\varphi)}{d\varphi}} = -\frac{\int_{\varphi_x^*}^{\varphi} \frac{d\Pi(\varphi)}{d\varphi} \, d\Gamma(\varphi)}{\int_{\varphi_x^*}^{\varphi} \frac{d\Pi(\varphi)}{d\varphi} \, d\Gamma(\varphi)} < 0,
\]

since the derivative of the profits of a firm with productivity \( \varphi \) with respect to the zero productivity cutoff \( \varphi^* \) using equation (39) is given by,

\[
\delta \frac{d\Pi(\varphi)}{d\varphi^*} = \sum_{i=1}^{j(\varphi)+1} \left[ 1 + (i-1) \tau^{\rho/(\rho-1)} \right]^{\rho-1} \int_{\varphi_x^{i-1}}^{\varphi_x^i} \rho^2 [\varphi l(w(\varphi))]^{\rho-1} \frac{\partial l(w(\varphi))}{\partial \varphi} \, d\varphi \\
- \sum_{i=1}^{j(\varphi)} \frac{\rho}{\varphi_x^i} f_x \frac{d\varphi_x^i}{d\varphi^*} - \frac{\rho}{\varphi^*} [\varphi^* l(w(\varphi^*))]^\rho < 0,
\]

\[(41)\]
where the second term follows from differentiating $\delta \Pi (\varphi)$ with respect to the export cutoff productivities $\varphi^x_i$ and using the export cutoff condition (17) to substitute for $[\varphi^x_i l (w (\varphi^x_i))]^\rho$.

The last term follows from noting that $\varphi^0_x = \varphi^*$. Note that $d\varphi^i_x / d\varphi^*$ in the second term is positive, since applying the implicit function theorem to the export cutoff condition (17) implies,

$$
\frac{d\varphi^i_x}{d\varphi^*} = - \frac{\rho}{\rho - 1} \frac{\partial l (w (\varphi^*_x))}{\partial \varphi^*_x} \left[ \varphi^*_x l (w (\varphi^*_x)) + \varphi^*_x \frac{\partial l (w (\varphi^*_x))}{\partial \varphi^*_x} \right] > 0,
$$

since $\partial l (w (\varphi^*_x)) / \partial \varphi > 0$ and $\partial l (w (\varphi^*_x)) / \partial \varphi^* < 0$ as the position of a firm in the wage offer distribution decreases if $\varphi^*$ increases. The same applies to $\partial l (w (\tilde{\varphi}^i_x)) / \partial \varphi^* < 0$ in the first term of equation (41). The derivative with respect to the number of total vacancies $M$ is given by,

$$
\delta \frac{d\Pi (\varphi)}{dM} = \sum_{i=1}^{j(\varphi)+1} \left[ 1 + (i - 1) \tau^{\rho-1} \right] \rho^{\rho-1} \left[ \varphi^i_x l (w (\varphi^i_x)) \right] \frac{\partial l (w (\varphi^*_i))}{\partial \varphi^*_i} d\varphi^*_i < 0,
$$

since $\partial l (w (\varphi^*_x)) / \partial M < 0$ as one can easily verify from equations (13). Note that, similar to the argument above, as $d\varphi^i_x / d\varphi^*$ in the second term is positive, one can easily verify from applying the implicit function theorem to the export cutoff condition (17) that $d\varphi^i_x / dM$ is positive. Thus, the free entry condition defines a decreasing relationship between the zero cutoff productivity $\varphi^*$ and the number of active firms $M$ in the market.

The same applies to a framework with convex vacancy creation costs but without on-the-job search, since only the labor input equation changes from (15) to (21), which implies that labor input is constant and thus independent of $\varphi^i_x$ and $\varphi^*$.

Applying the implicit function theorem to the zero cutoff profit condition (20) implies,

$$
\frac{d\varphi^*}{dM} = \frac{\rho [\varphi^*]^\rho [l (z)]^\rho - zl (z)}{\rho [\varphi^*]^\rho - 1 [l (z)]^\rho} \frac{1}{M} > 0,
$$

where Assumption 1, i.e., $\Pi'_w (z, 0|\varphi^*) \geq 0 \implies \rho [\varphi^*]^\rho [l (z)]^\rho > zl (z)$, ensures an increasing relation between the zero cutoff productivity $\varphi^*$ and the number of active firms $M$ in the market. This is true for a framework with and without on-the-job search, since the zero cutoff profit condition (20) is the same in both frameworks. Thus, a unique equilibrium in all frameworks with convex vacancy creation costs exists if unemployment benefits $z$ are low enough.

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G Existence Proof for an Equilibrium with Linear Vacancy Creation Costs

The zero cutoff productivity $\varphi^*$ is defined by the zero profit condition of the least productive firm. Using the profit equation (29), the zero profit condition can be stated as,

$$0 = (1 - \rho) \left[ \frac{z}{\rho} + \frac{c}{\rho} l(z, v(\varphi^*)) \right]^{\frac{1}{\rho - \rho}} \varphi^* \frac{v(\varphi^*)}{v(z, v(\varphi^*))} - f. \quad (44)$$

Using equation (23) or (24) to replace $v(\varphi^*) / l(z, v(\varphi^*))$, it follows that the zero cutoff profit condition defines an increasing relationship between $\varphi^*$ and $M\tilde{v}$.

In a framework without convex vacancy creation costs the free entry condition is given by integrating the profit equation (29) with $j(\varphi) = 0$ for non-exporting firms over the productivity range $[\varphi^*, \varphi_x]$ and $j(\varphi) = n$ for exporting firms over the productivity range $[\varphi_x, \infty)$ and by equating average profits to the entry costs $f_e$. Substituting $cv(\varphi^*) / l(z, v(\varphi^*))$ using the zero cutoff profit condition (44) implies,

$$\delta f_e = \int_{\varphi^*}^{\varphi_x} \frac{v(\varphi^*)}{v(\varphi^*)} d\Gamma(\varphi) + \int_{\varphi_x}^{\infty} \left[ 1 + n \pi \cdot \varphi^* \right] d\Gamma(\varphi) - \left[ 1 - \Gamma(\varphi^*) \right] f - nf_x [1 - \Gamma(\varphi_x)]. \quad (45)$$

The free entry condition, which is independent of $M\tilde{v}$, defines the zero cutoff productivity $\varphi^*$. Differentiating the right-hand side implies existence and uniqueness, i.e.,

$$\frac{\partial \delta \Pi_e}{\partial \varphi^*} = -\frac{1}{1 - \rho} \left[ \delta f_e + [1 - \Gamma(\varphi^*)] f + [1 - \Gamma(\varphi_x)] nf_x \right] < 0.$$

The total number of vacancies in the economy $M\tilde{v}$ is then given by the zero cutoff profit condition (44) where $v(\varphi^*) / l(z, v(\varphi^*))$ in case with and without on-the-job search is given by (using again equation (23) and (24), respectively),

$$\frac{v(\varphi^*)}{l(z, v(\varphi^*))} = \frac{[x + x + \lambda]^2}{\lambda (x + \lambda)} M\tilde{v}, \text{ and } \frac{v(\varphi^*)}{l(z, v(\varphi^*))} = \frac{x + x + \lambda}{\lambda} M\tilde{v}.$$

H Derivation of Wage Equation (25)

The optimality condition for vacancies (26) and wages,

$$\rho \left[ 1 + j(\varphi) \tau \cdot \pi \right] (1 - \rho) \varphi^o l(w(\varphi), v(\varphi))^{(\rho - 1)} - w(\varphi) = \frac{l(w(\varphi), v(\varphi))}{\partial w(\varphi)} \frac{\partial w(\varphi)}{\partial \varphi},$$

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imply the equality of marginal costs, i.e.,
\[
\left[ \frac{\partial l}{\partial w} \frac{\partial w}{\partial \varphi} \right] \frac{v(\varphi)}{l(w(\varphi), v(\varphi))} c = \frac{\partial w}{\partial \varphi} l(w(\varphi), v(\varphi)).
\] (46)

Similar to Appendix D, using equation (23) we define,
\[
\frac{l(w(\varphi), v(\varphi))}{v(\varphi)} = \lambda(\kappa + \phi) e^{T(\varphi)},
\] (47)

where,
\[
T(\varphi) = -\ln \left[ \frac{\kappa + \phi + \lambda [1 - F(w(\varphi))]^2}{\kappa + \phi + \lambda [1 - F(w(\varphi))]} \right],
\]
\[
T'(\varphi) = \frac{2\lambda f(w(\varphi))}{\kappa + \phi + \lambda [1 - F(w(\varphi))]} \frac{\partial w}{\partial \varphi}.
\]

Given the definition of \(l(w(\varphi), v(\varphi))\) in equation (23), i.e.,
\[
l(w(\varphi), v(\varphi)) = \lambda(\kappa + \phi) v(\varphi) \frac{1}{\kappa + \phi + \lambda [1 - F(w(\varphi))]^2},
\]
we get,
\[
\frac{\partial l}{\partial w} = l(w(\varphi), v(\varphi)) \frac{2\lambda f(w(\varphi))}{\kappa + \phi + \lambda [1 - F(w(\varphi))]}.
\]

Substituting this into equation (46) using \(T'(\varphi)\) and equation (47) allows us to write the differential equation for wages as follows,
\[
\frac{\partial w(\varphi)}{\partial \varphi} = \frac{cM\tilde{v}}{\lambda(\kappa + \phi)} \frac{T'(\varphi)}{e^{T(\varphi)}}.
\]

Integration gives,
\[
w(\varphi) = \frac{cM\tilde{v}}{\lambda(\kappa + \phi)} \int_{\varphi^*}^{\varphi} \frac{T'(\varphi)}{e^{T(\varphi)}} d\varphi + A
\]
\[
= \frac{cM\tilde{v}}{\lambda(\kappa + \phi)} \left[ e^{-T(\varphi^*)} - e^{-T(\varphi)} \right] + A
\]
\[
= c \left[ \frac{v(\varphi^*)}{l(0, v(\varphi^*))} - \frac{v(\varphi)}{l(w(\varphi), v(\varphi))} \right] + z,
\] (48)

where \(A = z\) follows from \(w(\varphi^*) = z\).

I Proof of Proposition 3

The export cutoff productivity \(\varphi_x^j\) is defined by \(\delta \Pi(w, j|\varphi_x^j) = \delta \Pi(w, j - 1|\varphi_x^j)\), where,
\[
\delta \Pi(w, j|\varphi) = \left[ 1 + j^\rho \frac{\rho}{\rho - 1} \right]^{(1-\rho)} [\varphi l(w(\varphi))] - w(\varphi) l(w(\varphi)) - f - jf_x.
\]

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Since profit maximization implies that the wage is continuous at \( \varphi_j^* \), and since the same wage implies that the number of workers employed by both types of firms are identical and given by \( l \left( w \left( \varphi_j^* \right) \right) \), we may write,

\[
\left[ 1 + j \tau \rho \right]^{(1-\rho)} \left[ \varphi_x^j l \left( w \left( \varphi_x^j \right) \right) \right]^{\rho} - w \left( \varphi_x^j \right) l \left( w \left( \varphi_x^j \right) \right) - j f_x
\]

\[
= \left[ 1 + (j - 1) \tau \rho \right]^{(1-\rho)} \left[ \varphi_x^j l \left( w \left( \varphi_x^j \right) \right) \right]^{\rho} - w \left( \varphi_x^j \right) l \left( w \left( \varphi_x^j \right) \right) - (j - 1) f_x.
\]

Thus, the export cutoff condition (17) can be derived,

\[
\left[ \varphi_x^j l \left( w \left( \varphi_x^j \right) \right) \right]^{\rho} = \frac{f_x}{\left[ 1 + j \tau \rho \right]^{(1-\rho)} - \left[ 1 + (j - 1) \tau \rho \right]^{(1-\rho)}}.
\]

The right-hand side of the last equation (and therefore the export cutoff productivity \( \varphi_x^j \)) is increasing in \( j \), i.e.,

\[
\left[ \varphi_x^j l \left( w \left( \varphi_x^j \right) \right) \right]^{\rho} - \left[ \varphi_x^{j-1} l \left( w \left( \varphi_x^{j-1} \right) \right) \right]^{\rho}
\]

\[
= f_x \frac{2 \left[ 1 + (j - 1) \tau \rho \right]^{(1-\rho)} - \left[ 1 + (j - 2) \tau \rho \right]^{(1-\rho)} - \left[ 1 + j \tau \rho \right]^{(1-\rho)}}{\left[ 1 + j \tau \rho \right]^{(1-\rho)} - \left[ 1 + (j - 1) \tau \rho \right]^{(1-\rho)}} > 0,
\]

where the last inequality follows from Jensen’s inequality, i.e.,

\[
\frac{1}{2} \left[ 1 + (j - 2) \tau \rho \right]^{(1-\rho)} + \frac{1}{2} \left[ 1 + j \tau \rho \right]^{(1-\rho)}
\]

\[
< \left[ \frac{1}{2} \left[ 1 + (j - 2) \tau \rho \right] + \frac{1}{2} \left[ 1 + j \tau \rho \right] \right]^{(1-\rho)}
\]

\[
= \left[ 1 + \frac{1}{2} (j - 2) + \frac{j}{2} \right] \left[ \tau \rho \right]^{(1-\rho)}
\]

\[
= \left[ 1 + (j - 1) \tau \rho \right]^{(1-\rho)}.
\]

J The Effects of Trade Liberalization on \( \varphi^* \) and \( M \) (or \( M\tilde{v} \))

The free entry condition always guarantees that the zero cutoff productivity \( \varphi^* \) and the total number of vacancies \( V \equiv M\tilde{v} \) in the case of convex vacancy creation costs and \( V \equiv M\tilde{v} \) in the case of linear vacancy creation costs, adjust such that average profits are equal to the entry costs \( f_e \). In order to show the effect of trade liberalization, we can thus use the implicit function theorem, i.e.,

\[
\Pi_e = f_e \iff \frac{\partial \Pi_e}{\partial x} + \sum_{i=1}^{n} \frac{d \Pi_e}{d \varphi_x^i} \frac{d \varphi_x^i}{dx} + \frac{d \Pi_e}{d \varphi^*} \frac{d \varphi^*}{dx} + \frac{d \Pi_e}{d V} \frac{d V}{dx} = 0,
\]
for every \( x \in \{\tau, f_x\} \).

**Convex Vacancy Creation Costs**

Note first that the zero cutoff profit condition (20) remains unchanged (compare Figure 2) if trade is liberalized. Thus, a change in the cutoff productivity \( \phi^* \) in response to trade liberalization must go along with a change in the total number of vacancies \( V \) in the same direction, since the zero cutoff profit (ZCP) condition defines an increasing relationship between \( \phi^* \) and \( V \) as shown in Appendix F, i.e., since \( d\phi^*/dM|_{ZCP} \) and therefore \( d\phi^*/dV|_{ZCP} > 0 \). Thus, the implicit function theorem implies,

\[
\frac{d\phi^*}{dx} = -\frac{\partial \Pi_e}{\partial \phi} \frac{\partial \phi}{\partial x} + \frac{\partial \Pi_e}{\partial f_x} \frac{d\phi}{dx} \quad \text{for every } x \in \{\tau, f_x\}. \tag{50}
\]

Average profits are given by,

\[
\Pi_e = \int_{\phi^*}^{\infty} \Pi(\phi) \gamma(\phi) \, d\phi \tag{51}
\]

\[
= \frac{1}{\delta} \int_{\phi^*}^{\infty} \sum_{i=1}^{n+1} \left[ 1 + (i-1) \tau^{\rho-1} \right]^{(1-\rho)} \int_{\phi^*}^{\phi_i} \rho \left( \frac{\bar{\phi} l(w(\bar{\phi}))}{\bar{\phi}} \right)^{\rho} \, d\bar{\phi} \right] \gamma(\phi) \, d\phi,
\]

where \( \phi^{n+1} = \phi \) and \( \phi^0 = \phi^* \). If on-the-job search is possible \( l(w(\phi)) \) is given by equation (15) and by equation (21) in the framework without on-the-job search. Before we apply the implicit function theorem let us investigate the single derivatives.

In order to use the implicit function theorem as stated in equation (50) consider the direct effect that the variables \( x \in \{\tau, f_x\} \) have on a firm’s profits. The shipping costs \( \tau \) enter profits directly and exporting fixed costs \( f_x \) only enter through the export cutoff productivities \( \phi^i_x \), i.e., (using equation 40),

\[
\delta \frac{\partial \Pi_e(\phi)}{\partial \tau} = -\sum_{i=1}^{j(\phi)+1} \rho (i-1) \tau^{\rho-1} \left[ 1 + (i-1) \tau^{\rho-1} \right]^{(1-\rho)} \int_{\phi^i_x-1}^{\phi^i_x} \rho \left( \frac{\bar{\phi} l(w(\bar{\phi}))}{\bar{\phi}} \right)^{\rho} \, d\bar{\phi} \right] < 0,
\]

\[
\delta \frac{\partial \Pi_e(\phi)}{\partial f_x} = 0.
\]

Thus, integrating the respective terms over \([\phi^*, \infty)\), weighted with \( \gamma(\phi) \), implies,

\[
\frac{\partial \Pi_e}{\partial \tau} < 0, \text{ and } \frac{\partial \Pi_e}{\partial f_x} = 0. \tag{52}
\]
The derivative of the profits of a firm with productivity \( \varphi \) with respect to the export cutoff productivities is given by,

\[
\frac{\delta}{\delta x} \left( \sum_{i=1}^{j(\varphi)} \frac{d\Pi(\varphi_i)}{d\varphi_i} \frac{d\varphi_i}{dx} \right) = \sum_{i=1}^{j(\varphi)} \left[ \left( 1 + (i - 1) \frac{\rho}{\tau - 1} \right)^{(1 - \rho)} - \left( 1 + i \frac{\rho}{\tau - 1} \right)^{(1 - \rho)} \right] \frac{\rho}{\varphi_i^\rho} \left( w(\varphi_i^\rho) \right)^\rho \frac{d\varphi_i}{dx} = - \sum_{i=1}^{j(\varphi)} \frac{\rho}{\varphi_i^\rho} \frac{df_i}{dx},
\]

where we used the fact that \( \varphi_{i+1} = \varphi \) and \( \varphi_0 = \varphi^* \) are not export cutoff productivities and that \( \left[ \varphi_i^\rho \left( w(\varphi_i^\rho) \right) \right]^\rho \) in the second line can be substituted using the export cutoff condition \((17)\). Applying the implicit function theorem to the export cutoff equation \((17)\) shows, i.e.,

\[
\frac{d\varphi_i^\rho}{df} = \frac{\varphi_i^\rho}{\left( 1 + \frac{\varphi_i^\rho}{l\left( w(\varphi_i^\rho) \right)} \frac{\partial l}{\partial \varphi_i^\rho} \right)^\rho} > 0,
\]

\[
\frac{d\varphi_i^\rho}{d\tau} = \frac{\varphi_i^\rho}{\left[ 1 + \frac{\varphi_i^\rho}{l\left( w(\varphi_i^\rho) \right)} \frac{\partial l}{\partial \varphi_i^\rho} \right]^{(1 - \rho)} - \left[ 1 + (j - 1) \frac{\rho}{\tau - 1} \right]^{(1 - \rho)} \left[ 1 + \frac{\varphi_i^\rho}{l\left( w(\varphi_i^\rho) \right)} \frac{\partial l}{\partial \varphi_i^\rho} \right]} > 0,
\]

where the last inequality follows from,

\[
\left[ 1 + i \frac{\rho}{\tau - 1} \right]^{(1 - \rho)} - \left[ 1 + (i - 1) \frac{\rho}{\tau - 1} \right]^{(1 - \rho)} > 0,
\]

and noting that \( \left[ 1 + j \frac{\rho}{\tau - 1} \right]^{(1 - \rho)} - \left[ 1 + (j - 1) \frac{\rho}{\tau - 1} \right]^{(1 - \rho)} > 0 \), as otherwise \( \varphi_i^\rho l\left( w(\varphi_i^\rho) \right)^\rho \) would be negative according to equation \((17)\). Note that the sign of \( d\varphi_i^\rho /df \) and \( d\varphi_i^\rho /d\tau \) is the same irrespective of whether the framework is with or without on-the-job search, since the sign does not depend on whether \( \partial l\left( w(\varphi_i^\rho) \right) /\partial \varphi_i^\rho > 0 \) as in the framework with on-the-job search or \( \partial l\left( w(\varphi_i^\rho) \right) /\partial \varphi_i^\rho = 0 \) as in the framework without on-the-job search.
Integrating the respective terms over \([\varphi^*, \infty)\), weighted with \(\gamma(\varphi)\), implies,

\[
\sum_{i=1}^{n} \frac{d\Pi_e}{d\varphi^i_x} \frac{d\varphi^i_x}{df_x} = \int_{\varphi^*}^{\infty} \sum_{i(\varphi)=1}^{j(\varphi)} \frac{d\Pi_e}{d\varphi^i_x} \frac{d\varphi^i_x}{df_x} \gamma(\varphi) \, d\varphi < 0, \quad \text{and} \quad (53)
\]

\[
\sum_{i=1}^{n} \frac{d\Pi_e}{d\varphi^i_x} \frac{d\varphi^i_x}{d\tau} = \int_{\varphi^*}^{\infty} \sum_{i(\varphi)=1}^{j(\varphi)} \frac{d\Pi_e}{d\varphi^i_x} \frac{d\varphi^i_x}{d\tau} \gamma(\varphi) \, d\varphi < 0. \quad (54)
\]

The effect of a change in the potential trading partners \(n\) does not alter the export cutoffs as equation (17) is independent of \(n\).

The derivative of the profits of a firm with productivity \(\varphi\) with respect to the zero productivity cutoff \(\varphi^*\) and \(M\) (and therefore \(V\)) is given by equations (41) and (42) in Appendix F, respectively. Integrating the respective derivatives over \([\varphi^*, \infty)\) weighted with \(\gamma(\varphi)\) implies,

\[
\frac{d\Pi_e}{d\varphi^*} + \frac{d\Pi_e}{dV} \frac{dV}{d\varphi^*} < 0. \quad (55)
\]

Inserting equations (52), (53), (54), and (55) into formula (50) implies that trade liberalization (\(\tau \downarrow, f_x \downarrow\)) increases the zero cutoff productivity \(\varphi^*\) and thus also the total number of vacancies \(V\) in the economy.

### Linear Vacancy Creation Costs

We can use the free entry condition (45) to derive the effect of trade liberalization on the cutoff productivity \(\varphi^*\). Changing the variable of integration from \(\varphi\) to \(\Gamma(\varphi) \equiv [\Gamma(\varphi) - \Gamma(\varphi^*)]/[1 - \Gamma(\varphi^*)] = 1 - (\varphi^*/\varphi)^\gamma\) implies the following equation,

\[
\delta f_e = f [1 - \Gamma(\varphi^*)] \int_{0}^{\Gamma(\varphi_x)} \left[1 - \Gamma(\varphi_x)^{-\frac{1}{\gamma}}\right]^{-\frac{1}{\gamma} + \frac{\rho}{1 - \gamma}} \Gamma(\varphi_x) \, d\Gamma
\]

\[
+ f [1 - \Gamma(\varphi^*)] \left[1 + n \tau \frac{\rho}{1 - \gamma} \right] \int_{\Gamma(\varphi_x)}^{1} \left[1 - \Gamma(\varphi_x)^{-\frac{1}{\gamma}}\right]^{-\frac{1}{\gamma} + \frac{\rho}{1 - \gamma}} \Gamma(\varphi_x) \, d\Gamma
\]

\[
- [1 - \Gamma(\varphi^*)] f - [1 - \Gamma(\varphi^*)] \left[1 - \Gamma(\varphi_x)\right] nf_x.
\]

Taking the derivatives of the right-hand side of equation (56) with respect to \(\varphi^*\), \(f_x\), and \(\tau\),
i.e.,

\[
\frac{\partial \delta \Pi_e}{\partial \varphi^*} = -\frac{\gamma (\varphi^*)}{[1 - \Gamma (\varphi^*)]} \delta \Pi_e + f [1 - \Gamma (\varphi^*)] \frac{\partial \tilde{\Gamma} (\varphi_x)}{\partial \varphi^*} n \left[ \frac{f_x}{f} - \tau^{\frac{\rho}{\rho - 1}} \left[ 1 - \tilde{\Gamma} (\varphi_x) \right]^{-\frac{1}{\gamma} \frac{\rho}{\rho - 1}} \right]
\]

\[
= -\frac{\gamma (\varphi^*)}{[1 - \Gamma (\varphi^*)]} \delta \Pi_e < 0,
\]

\[
\frac{\partial \delta \Pi_e}{\partial f_x} = -f [1 - \Gamma (\varphi^*)] n \tau^{\frac{\rho}{\rho - 1}} \left[ 1 - \tilde{\Gamma} (\varphi_x) \right]^{-\frac{1}{\gamma} \frac{\rho}{\rho - 1}} \frac{\partial \tilde{\Gamma} (\varphi_x)}{\partial f_x}
\]

\[
+ [1 - \Gamma (\varphi^*)] \frac{\partial \tilde{\Gamma} (\varphi_x)}{\partial f_x} n f_x - [1 - \Gamma (\varphi^*)] \left[ 1 - \tilde{\Gamma} (\varphi_x) \right] n
\]

\[
= - [1 - \Gamma (\varphi^*)] \frac{1}{\tau^{\gamma}} \left[ \frac{f_x}{f_x} \right]^{\frac{\tau - \rho}{\rho}} n < 0,
\]

\[
\frac{\partial \delta \Pi_e}{\partial \tau} = -f [1 - \Gamma (\varphi^*)] n \tau^{\frac{\rho}{\rho - 1}} \left[ 1 - \tilde{\Gamma} (\varphi_x) \right]^{-\frac{1}{\gamma} \frac{\rho}{\rho - 1}} \frac{\partial \tilde{\Gamma} (\varphi_x)}{\partial \tau}
\]

\[
- f [1 - \Gamma (\varphi^*)] \frac{\rho}{1 - \rho} n \tau^{\frac{1}{\rho - 1}} \int \frac{1}{\tilde{\Gamma} (\varphi_x)} \left[ 1 - \tilde{\Gamma} \right]^{-\frac{1}{\gamma} \frac{\rho}{\rho - 1}} d\tilde{\Gamma}
\]

\[
+ [1 - \Gamma (\varphi^*)] \frac{\partial \tilde{\Gamma} (\varphi_x)}{\partial \tau} n f_x
\]

\[
= -f [1 - \Gamma (\varphi^*)] \frac{\rho}{1 - \rho} n \tau^{\frac{1}{\rho - 1}} \int \frac{1}{\tilde{\Gamma} (\varphi_x)} \left[ 1 - \tilde{\Gamma} \right]^{-\frac{1}{\gamma} \frac{\rho}{\rho - 1}} d\tilde{\Gamma} < 0,
\]

since according to equation \([32]\) we know that,

\[
\left[ 1 - \tilde{\Gamma} (\varphi_x) \right]^{\frac{1}{\gamma}} = \frac{\varphi^*}{\varphi_x} = \frac{1}{\tau} \left[ \frac{f_x}{f_x} \right]^{\frac{1 - \rho}{\rho}}.
\]

Using the implicit function theorem implies that the cutoff productivity \(\varphi^*\) increases as trade is liberalized, i.e., \(f_x \downarrow\) and \(\tau \downarrow\). The inequalities in the last two derivatives follow from,

\[
\frac{\partial \tilde{\Gamma} (\varphi_x)}{\partial f_x} = \frac{\partial}{\partial f_x} \left[ 1 - \left( \frac{\varphi^*}{\varphi_x} \right)^\gamma \right]
\]

\[
= \gamma \frac{1 - \rho}{\rho} \frac{1}{\tau^{\gamma}} \left[ \frac{f_x}{f_x} \right]^{\gamma \frac{1 - \rho}{\rho}} \frac{1}{f_x} > 0,
\]

\[
\frac{\partial \tilde{\Gamma} (\varphi_x)}{\partial \tau} = \gamma \frac{1}{\tau^{\gamma}} \left[ \frac{f_x}{f_x} \right]^{\gamma \frac{1 - \rho}{\rho}} > 0,
\]

where,

\[
\left( \frac{\varphi^*}{\varphi_x} \right)^\gamma = \frac{1}{\tau^{\gamma}} \left[ \frac{f_x}{f_x} \right]^{\gamma \frac{1 - \rho}{\rho}},
\]

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is obtained by dividing the zero cutoff profit condition (44) by the export cutoff condition (30).

Thus, the increase in the total number of vacancies \( M \tilde{v} \) follows from the fact that the zero cutoff profit condition (44), which remains unchanged if trade is liberalized, defines an increasing relationship between \( \varphi^* \) and \( M \tilde{v} \).

The results hold for both cases with and without on-the-job search, since the cutoff productivity \( \varphi^* \) is the same in both cases and the zero cutoff profit condition (44) has the same properties.

**K  Proof of Proposition 4**

**Convex Vacancy Creation Costs**

Let us first analyse the case with on-the-job search. Combining the export cutoff condition (17) with the zero profit condition (20) implies,

\[
G = \left[ \frac{\varphi^* l(z)}{\varphi_x l \left( \frac{w_y}{\varphi_x} \right)} \right]^\rho - \frac{f_{fx}}{f_x} \left[ 1 + j \tau^\rho \right]^{(1-\rho)} - \left[ 1 + (j-1) \tau^\rho \right]^{(1-\rho)}.
\]

Using,

\[
1 - \tilde{\Gamma} \left( \varphi_x \right) = \left( \frac{\varphi_x}{\varphi_x} \right)^\gamma,
\]

and the labor input according to equation (15) implies,

\[
\left[ \frac{\varphi^* l(z)}{\varphi_x l \left( \frac{w_y}{\varphi_x} \right)} \right]^\rho = \left[ 1 - \tilde{\Gamma} \left( \varphi_x \right) \right]^\gamma \left[ \frac{x + \phi + \lambda \left[ 1 - \tilde{\Gamma} \left( \varphi_x \right) \right]}{x + \phi + \lambda} \right]^{2\rho}.
\]

Differentiating \( G \) with respect to the fraction of firms that export to at least \( j \) countries \( \tilde{\Gamma} \left( \varphi_x \right) \), and with respect to \( f_x \) and \( \tau \),

\[
G'_{\tilde{\Gamma}(\varphi_x)} = -\frac{\rho}{\gamma} \left[ 1 - \tilde{\Gamma} \left( \varphi_x \right) \right]^\frac{\rho}{\gamma} \left[ \frac{x + \phi + \lambda \left[ 1 - \tilde{\Gamma} \left( \varphi_x \right) \right]}{x + \phi + \lambda} \right]^{2\rho} - \left[ 1 - \tilde{\Gamma} \left( \varphi_x \right) \right]^\gamma 2\rho \left[ \frac{x + \phi + \lambda \left[ 1 - \tilde{\Gamma} \left( \varphi_x \right) \right]}{x + \phi + \lambda} \right]^{2\rho-1} \frac{\lambda}{x + \phi + \lambda} < 0,
\]

\[
G'_{f_x} = \frac{f}{(f_x)^2} \left[ 1 + j \tau^\rho \right]^{(1-\rho)} - \left[ 1 + (j-1) \tau^\rho \right]^{(1-\rho)} > 0,
\]

\[
G'_{\tau} = \frac{f}{(f_x)^2 \tau^\rho} \left[ j \left[ 1 + j \tau^\rho \right]^{-\rho} - (j-1) \left[ 1 + (j-1) \tau^\rho \right]^{-\rho} \right] > 0,
\]

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where the last inequality follows from,

\[
\begin{align*}
    &j \left[ 1 + j \tau^{\rho/(\rho-1)} \right]^{-\rho} > (j - 1) \left[ 1 + (j - 1) \tau^{\rho/(\rho-1)} \right]^{-\rho}, \\
    &1 + \frac{1}{j - 1} > \left[ 1 + \frac{1}{\tau^{\rho/(\rho-1)} + (j - 1)} \right]^\rho \quad \text{for any } \rho \in (0, 1).
\end{align*}
\]

The implicit function theorem therefore implies that the fraction of firms that export to at least \(j\) export destinations increases in response to trade liberalization, i.e., \(f_x \downarrow\) and \(\tau \downarrow\).

Let us now determine the derivatives \(d^2\tilde{\Gamma}(\varphi^j_x)/df_xdj\) and \(d^2\tilde{\Gamma}(\varphi^j_x)/d\tau dj\). Since \(G''_{\tilde{\Gamma}(\varphi^j_x)}\) is independent of \(j\) we get,

\[
\begin{align*}
    &d^2\tilde{\Gamma}(\varphi^j_x)/df_xdj = -G''_{\tilde{\Gamma}(\varphi^j_x)} < 0 \quad \text{and} \quad d^2\tilde{\Gamma}(\varphi^j_x)/d\tau dj = -G''_{\tilde{\Gamma}(\varphi^j_x)} < 0.
\end{align*}
\]

This implies that the fraction of firms exporting to \(j\) or less export destinations increases at an increasing rate if trade is liberalized and that the fraction of firms exporting to \(j\) or more export destinations increases at a decreasing rate. Put differently, the fraction of domestic firms exporting to \(j \in \{1, ..., n-1\}\) export destinations decrease with trade liberalization, while the fraction of firms exporting to \(n\) destinations increases. The above results follow from,

\[
G''_{f_x,j} = \frac{f}{(f_x)^2} (1 - \rho) \tau^{-\rho-1} \left[ 1 + j \tau^{-\rho} \right]^{-\rho} \left[ 1 + (j - 1) \tau^{-\rho} \right]^{-\rho} < 0,
\]

because,

\[
\begin{align*}
    &\left[ 1 + j \tau^{-\rho} \right]^{-\rho} < \left[ 1 + (j - 1) \tau^{-\rho} \right]^{-\rho}, \\
    &1 < \left[ 1 + \frac{\tau^{\rho/(\rho-1)}}{1 + (j - 1) \tau^{\rho/(\rho-1)}} \right]^\rho \quad \text{for any } \rho \in (0, 1),
\end{align*}
\]

and from,

\[
G''_{\tau dj} = \frac{f}{(f_x)^2} \rho \tau^{-\rho-1} \left[ 1 + j \tau^{-\rho} \right]^{-\rho} \left[ \frac{\rho}{\tau^{-\rho} + j} \right]^{-\rho} - \frac{f}{(f_x)^2} \rho \tau^{-\rho-1} \left[ 1 + (j - 1) \tau^{-\rho} \right]^{-\rho} \left[ \frac{\rho}{\tau^{-\rho} + (j - 1)} \right]^{-\rho-1} < 0,
\]

and from,

\[
G''_{\tau j} = -\frac{f}{(f_x)^2} \rho \tau^{-\rho-1} \left[ 1 + j \tau^{-\rho} \right]^{-\rho} \left[ \frac{\rho}{\tau^{-\rho} + j} \right]^{-\rho} - \frac{f}{(f_x)^2} \rho \tau^{-\rho-1} \left[ 1 + (j - 1) \tau^{-\rho} \right]^{-\rho} \left[ \frac{\rho}{\tau^{-\rho} + (j - 1)} \right]^{-\rho-1} < 0,
\]

and from,
because,
\[
\left[1 + j \tau^{\rho/(\rho-1)}\right]^{-(\rho-1)} - \rho \tau^{\rho/(\rho-1)} j \left[1 + j \tau^{\rho/(\rho-1)}\right]^{-(\rho-1)} < \left[1 + (j - 1) \tau^{\rho/(\rho-1)}\right]^{-(\rho-1)} - \rho \tau^{\rho/(\rho-1)} (j - 1) \left[1 + (j - 1) \tau^{\rho/(\rho-1)}\right]^{-(\rho-1)}
\]
\[
\left[1 + j \tau^{\rho/(\rho-1)}\right]^{-(\rho-1)} \left[1 - \rho j \tau^{\rho/(\rho-1)}\right] < \left[1 + (j - 1) \tau^{\rho/(\rho-1)}\right]^{-\rho} \left[1 - \rho (j - 1) \tau^{\rho/(\rho-1)}\right]
\]
\[
\frac{1 + (1 - \rho) j \tau^{\rho/(\rho-1)}}{1 + (1 - \rho) (j - 1) \tau^{\rho/(\rho-1)}} < \left[\frac{1 + j \tau^{\rho/(\rho-1)}}{1 + (j - 1) \tau^{\rho/(\rho-1)}}\right]^\rho + 1 \text{ for any } \rho \in (0, 1).
\]
The latter follows because,
\[
\frac{1 + (1 - \rho) j \tau^{\rho/(\rho-1)}}{1 + (1 - \rho) (j - 1) \tau^{\rho/(\rho-1)}} < \frac{1 + j \tau^{\rho/(\rho-1)}}{1 + (j - 1) \tau^{\rho/(\rho-1)}} < \left[\frac{1 + j \tau^{\rho/(\rho-1)}}{1 + (j - 1) \tau^{\rho/(\rho-1)}}\right]^\rho + 1.
\]
Note that this result holds also for the case without on-the-job search, which implies according to equation (21) that labor input is identical across firms, i.e., \( l \left( w \left( \varphi_j^x \right) \right) = l \left( z \right) \). Hence we get,
\[
\left[\frac{\varphi^* l \left( z \right)}{\varphi_j^x l \left( w \left( \varphi_j^x \right) \right)}\right]^{\rho} = \left[1 - \tilde{\Gamma} \left( \varphi_j^x \right)\right]^{\frac{\rho}{\gamma}},
\]
which implies \( G'_{\tilde{\Gamma}(\varphi_j^x)} < 0 \). The remaining results follow from above.

The trade pattern, which exists according to Proposition 3 in all models with convex vacancy creation costs, also remains if trade is fully liberalized, i.e., \( f_x \to f, \tau \to 1 \). This follows from the fact that,
\[
\lim_{f_x \to f, \tau \to 1} \frac{f}{f_x} \left[\left[1 + j \tau^{\rho/(\rho-1)}\right]^{(1-\rho)} - \left[1 + (j - 1) \tau^{\rho/(\rho-1)}\right]^{(1-\rho)}\right]
\]
\[= \left[\left[1 + j\right]^{(1-\rho)} - \left[1 + (j - 1)\right]^{(1-\rho)}\right].
\]

**Linear Vacancy Creation Costs**

Combining the export-cut-off condition (30) with the zero profit condition (44) implies,
\[
\frac{\varphi^*}{\varphi_j^x} = \frac{1}{\tau} \left[\frac{f}{f_x}\right]^{1-\rho}.\]

Thus, with linear vacancy creation costs the fraction of exporting firms \( \left( \varphi^* / \varphi_j^x \right) \gamma \) increases if trade is liberalized, i.e., \( \tau \downarrow \) and \( f_x \downarrow \). Furthermore, the model with linear vacancy creation
costs predicts that all firms that start to export will export to all \( n \) destination countries. If trade is fully liberalized, i.e., \( f_x \to f, \tau \to 1 \), the model predicts that all firms will export to all countries. This follows from \( \lim_{{f_x \to f, \tau \to 1}} (\varphi^*/\varphi_x) = 1 \).

L Proof of Propositions 6 and 7

We can concentrate on the frameworks with on-the-job search, since all frameworks without on-the-job search have wages equal to the reservation wage of \( z \).

Linear Vacancy Creation Costs

Consider the wage equation (25), i.e.,

\[
    w(\varphi) = c \left( \frac{v(\varphi^*)}{l(z, v(\varphi^*))} - \frac{v(\varphi)}{l(w(\varphi), v(\varphi))} \right) + z.
\]

Using equations (13) and (16) we can write,

\[
    \tilde{\Gamma}(\varphi) = 1 - \left( \frac{\varphi^*}{\varphi} \right)^\gamma = \frac{(\kappa + \phi + \lambda) G(w(\varphi))}{\kappa + \phi + \lambda G(w(\varphi))},
\]

which can be substituted into \( v(\varphi) / l(w(\varphi), v(\varphi)) \) given by equation (23) to obtain,

\[
    \frac{v(\varphi)}{l(w(\varphi), v(\varphi))} = \left[ \frac{\kappa + \phi + \lambda}{\kappa + \phi + \lambda G(w(\varphi))} \right]^2 \frac{(\kappa + \phi) M\tilde{v}}{\lambda}.
\]

Substituting into the wage equation allows us to write the wage as a function of the quantil of the wage earnings distribution \( G \), i.e.,

\[
    w(G) = c \left[ 1 - \left( \frac{\kappa + \phi}{\kappa + \phi + \lambda G} \right)^2 \left[ \frac{\kappa + \phi + \lambda}{\kappa + \phi} \right]^2 \frac{(\kappa + \phi) M\tilde{v}}{\lambda} \right] + z.
\]

Trade liberalization \((\tau \downarrow, f_x \downarrow)\) has only an indirect effect on wages via the number of vacancies \( M\tilde{v} \) in the economy. Since trade liberalization increases the number of vacancies as shown in Appendix J, it follows that wages increase with trade liberalization (except for the firm at the cutoff productivity, which pays the reservation wage). Furthermore, wages at higher quantiles increase more, since,

\[
    \frac{\partial^2 w(G)}{\partial (M\tilde{v}) \partial G} = \frac{2c (\kappa + \phi)}{\kappa + \phi + \lambda G} \left[ \frac{\kappa + \phi + \lambda}{\kappa + \phi} \right]^2 > 0.
\]

In order to prove the effect on wage inequality we use Lorenz dominance. In order to be able to derive analytical results we normalize unemployment benefit to zero, i.e., \( z = 0 \), in the
following. Wage inequality according to Lorenz dominance increases with trade liberalization \((\tau \downarrow, f_x \downarrow)\), if,
\[
\frac{\partial}{\partial x} \int_0^G w(G') dG' > 0 \text{ for all } G \in [0, 1], \text{ where } x \in \{\tau, f_x\}.
\]
Substituting \(w(G)\) implies that the degree of wage inequality is independent of trade liberalization, i.e.,
\[
\frac{\int_0^G w(G') dG'}{\int_0^1 w(G') dG'} = \frac{\int_0^G \left[1 - \left[\frac{x + \phi}{\tau + \phi + x \tau} + \frac{x + \phi}{\tau + \phi + x \tau} \right]^2\right] dG'}{\int_0^1 \left[1 - \left[\frac{x + \phi}{\tau + \phi + x \tau} + \frac{x + \phi}{\tau + \phi + x \tau} \right]^2\right] dG'}.
\]
This also holds if we assume that unemployment benefits are proportional to the average wages, i.e., \(z = b \int_0^1 w(G') dG'\),
\[
\frac{\int_0^G w(G') dG'}{\int_0^1 w(G') dG'} = \frac{\int_0^G \left[1 - \left[\frac{x + \phi}{\tau + \phi + x \tau} + \frac{x + \phi}{\tau + \phi + x \tau} \right]^2\right] dG' - b \int_0^1 \left[1 - \left[\frac{x + \phi}{\tau + \phi + x \tau} + \frac{x + \phi}{\tau + \phi + x \tau} \right]^2\right] dG'.
\]

**Convex Vacancy Creation Costs**

Let us start with taking the first order condition for wages, i.e.,
\[
1 = \left[1 + j(\varphi) T^{\rho} \right]^{(1-\rho)} \varphi(\rho) \rho l(\varphi)^{\rho-1} - w(\varphi) \frac{\partial l(w(\varphi))}{\partial l(w(\varphi))} \frac{1}{l(w(\varphi))}.
\]
Using the fact that wages increase with \(G\) gives,
\[
\frac{\partial w(G)}{\partial G} = \left[1 + j(G) T^{\rho} \right]^{(1-\rho)} \varphi(G)^\rho \rho l(G)^{\rho-1} - w(G) \frac{\partial l(G)}{\partial G} \frac{1}{l(G)}.
\]
Define,
\[
T(G) = \ln[l(G)] \quad \text{and} \quad T'(G) = \frac{\partial l(G)}{\partial G} \frac{1}{l(G)}.
\]
Substitution simplifies the above differential equation to,
\[
\frac{\partial w(G)}{\partial G} e^{T(\varphi)} + w(G) e^{T(\varphi) T'(G)} = \left[1 + j(G) T^{\rho} \right]^{(1-\rho)} \varphi(G)^\rho \rho l(G)^{\rho} T'(G).
\]
Any solution to this differential equation has to satisfy,
\[
w(G) e^{T(\varphi)} = \int_{G_2}^{G} \left[1 + j(G) T^{\rho} \right]^{(1-\rho)} \varphi(G)^\rho \rho l(G)^{\rho} T'(G') dG' + A,
\]
or
\[
w(G) = \frac{1}{l(G)} \sum_{i=0}^{j(G)} \int_{G_2}^{G_{i+1}} \left[1 + i T^{\rho} \right]^{(1-\rho)} \rho \left[\varphi(G') l(G')\right]^\rho T'(G') dG' + z, \quad (57)
\]
where $G_{x}^{j(G)+1} = G$ and $G_{x}^{0} = 0$ and $w (0) = z$.

We also know that using equations (13), (15) and (16) we can write,

$$l (G) = \frac{\lambda}{\lambda + \phi} \left[ \frac{x + \phi + \lambda G}{\lambda + \phi + \lambda} \right]^{2} \frac{1}{M},$$

$$T_{G}^{'} (G) = \frac{2\lambda}{\lambda + \phi + \lambda G}.$$  

Using the Pareto distribution $F (G) = 1 - (\varphi^{*}/\varphi)^{\gamma}$ we get,

$$\varphi (G) = \varphi^{*} \left( \frac{(x + \phi) (1 - G)}{\lambda + \phi + \lambda G} \right)^{-\frac{1}{\gamma}}.$$  

The quantile $G_{x}^{i}$ at which firms export to $i$ countries is implicitly defined by combining the export cutoff condition (17) with the zero profit condition (20), i.e.,

$$H = \left( \frac{x + \phi}{\lambda + \phi + \lambda G_{x}^{i}} \right)^{2} \frac{1}{\rho} \left( \frac{1 - \rho}{\rho - 1} \right) \left( \frac{1 - G_{x}^{i}}{\lambda + \phi + \lambda G_{x}^{i}} \right)^{-\frac{1}{\gamma}} < 0,$$

since,

$$\frac{\partial H}{\partial G_{x}^{i}} = - (x + \phi)^{2} \gamma \frac{\rho}{\gamma} \left( 1 - G_{x}^{i} \right)^{-1} \left( 2 \rho + \frac{\rho}{\gamma} \right) \lambda \left( x + \phi + \lambda G_{x}^{i} \right)^{-1} \frac{1}{\rho} \left( \frac{1 - \rho}{\rho - 1} \right) \left( \frac{1 - G_{x}^{i}}{\lambda + \phi + \lambda G_{x}^{i}} \right)^{-\frac{1}{\gamma}},$$

$$\frac{\partial H}{\partial f_{x}} = \frac{f}{f_{x}} \left[ \left( 1 + i \tau^{\rho - 1} \right)^{1 - \rho} \right] \left( 1 + (i - 1) \tau^{\rho - 1} \right) > 0,$$

$$\frac{\partial H}{\partial \tau} = \frac{f}{f_{x}} \rho \tau^{\rho - 1} \left( 1 + i \tau^{\rho - 1} \right)^{-\rho} \left( 1 + (i - 1) \tau^{\rho - 1} \right)^{-\rho} > 0,$$

since,

$$i \left[ 1 + i \tau^{\rho/\left( \rho - 1 \right)} \right]^{-\rho} > (i - 1) \left[ 1 + (i - 1) \tau^{\rho/\left( \rho - 1 \right)} \right]^{-\rho},$$  

$$1 + \frac{1}{i - 1} > \left[ 1 + \frac{1}{\tau^{\rho/\left( \rho - 1 \right)} + (i - 1)} \right]^{\rho} \text{ for any } \rho \in (0, 1).
Differentiating equation (57) implies that wages increase with trade liberalization ($\tau \downarrow, f_x \downarrow$),

$$
\frac{\partial w (G)}{\partial f_x} = \frac{1}{M} \frac{\partial M}{\partial f_x} w (G) \\
- \frac{1}{l (G)} \left( \sum_{i=1}^{j(G)} \left[ 1 + i \tau^{\frac{\rho}{\rho - 1}} \right]^{(1 - \rho)} - 1 \right) \left( 1 - \rho \right) M \frac{\partial M}{\partial f_x} w (G) \\
\times \rho \left[ \varphi (G_x) \ell (G_x) \right]^{\rho} T' (G_x) \frac{\partial G_x}{\partial f_x} \\
< 0,
$$

$$
\frac{\partial w (G)}{\partial \tau} = \frac{1}{M} \frac{\partial M}{\partial \tau} w (G) \\
- \frac{1}{l (G)} \sum_{i=0}^{j(G)} \left( \rho i \tau^{\frac{\rho}{\rho - 1}} \right) \left[ 1 + i \tau^{\frac{\rho}{\rho - 1}} \right]^{(1 - \rho)} \rho \left[ \varphi (G') \ell (G') \right]^{\rho} T' (G') dG' \\
- \frac{1}{l (G)} \sum_{i=1}^{j(G)} \left[ 1 + i \tau^{\frac{\rho}{\rho - 1}} \right]^{(1 - \rho)} - 1 \right) \left( 1 - \rho \right) M \frac{\partial M}{\partial \tau} w (G) \\
\times \rho \left[ \varphi (G_x) \ell (G_x) \right]^{\rho} T' (G_x) \frac{\partial G_x}{\partial \tau} \\
< 0,
$$

since the number of active firms $M$ decreases with $f_x$ and $\tau$ as shown in Appendix J. Thus, all wages increase except for the reservation wage $z$.

In order to prove the effect on wage inequality we use Lorenz dominance. In order to be able to derive analytical results we normalize unemployment benefit to zero, i.e., $z = 0$, in the following. Wage inequality according to Lorenz dominance increases with trade liberalization ($\tau \downarrow, f_x \downarrow$), if,

$$
\frac{\partial}{\partial x} \int_0^G w (\tilde{G}) d\tilde{G} > 0 \text{ for all } G \in [0, 1), \text{ where } x \in \{\tau, f_x\}.
$$

Since we can write,

$$
w (G) = \frac{1}{l (G)} \sum_{i=0}^{j(G)} \int_{G_x}^{G_x^{i+1}} \left[ 1 + i \tau^{\frac{\rho}{\rho - 1}} \right]^{(1 - \rho)} \rho \left[ \varphi (G') \ell (G') \right]^{\rho} T' (G') dG' \\
= \rho [\varphi^*]^{\rho} M^{(1 - \rho)} (\kappa + \phi) \left( 1 - \rho - \frac{\rho}{\rho - 1} \right) [\kappa + \phi + \lambda]^{2(1 - \rho)} (\lambda)^{\rho - 1} R (G),
$$
where,

\[
R(G) = \frac{2\lambda}{[\kappa + \phi + \lambda G]^2} \sum_{i=0}^{j(G)} \int_{G_i}^{G_i+1} \left[ 1 + i\gamma \frac{G'}{G} \right]^{1-\rho} \left( 1 - G' \right)^{-\frac{\phi}{2}} \left( \kappa + \phi + \lambda G' \right)^{2\rho - 1 + \frac{\phi}{2}} dG'.
\]

The Lorenz dominance criterion is therefore given by,

\[
\frac{\int_0^G R(G) \, dG}{\int_0^1 R(G) \, dG}.
\]

Since \(R(G)\) changes with trade liberalization (not proportionally) we know that trade liberalization affects wage inequality. This also holds if we assume that unemployment benefits are proportional to the average wages, i.e., \(z = b \int_0^1 w(G') \, dG'\),

\[
\frac{\int_0^G R(G) \, dG - b \int_0^1 R(G) \, dG}{\int_0^1 R(G) \, dG - b \int_0^1 R(G) \, dG}.
\]

Under autarky we have,

\[
R_a(G) = \frac{2\lambda}{[\kappa + \phi + \lambda G]^2} \int_0^G \left( 1 - G' \right)^{-\frac{\phi}{2}} \left( \kappa + \phi + \lambda G' \right)^{2\rho - 1 + \frac{\phi}{2}} dG'.
\]

This implies \(R(G) \geq R_a(G)\) for all \(G\) and \(R(G) > R_a(G)\) for all \(G \geq G_a^1\). This implies that wage inequality in an open economy (even under free trade) is higher than under autarky.