Employment Protection and the Market for Innovations

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Abstract

We study the effects of employment protection taking into account that firms can – as a second best response – invest in R&amp;D or buy new technologies in order to restore their productivity. To do so we develop an equilibrium matching model with an imperfect labor and innovation market. If employment protection is introduced, firms’ willingness to pay for product or process innovations increases. This shifts economic activity towards firms specializing in process and product innovation and triggers entry of new start-ups. We calibrate our model to match aggregate US labor and product market statistics and show that our model generates the negative impact of wrongful dismissal laws in the US on productivity and the positive effect on the number of innovations and firms found by the literature.

Keywords: Employment protection, firing costs, innovations, patents, productivity, market imperfections

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1. Introduction

Employment protection legislation (EPL) is thought to ensure workers against temporary productivity shocks. While most negative productivity shocks to a firm are exogenous, like a drop in demand due to changes in taste or an increase in competition due to new production technologies of competitors, positive productivity shocks are usually the result of process or product innovation and are hence endogenous. Product or process innovation can either be done within a firm through own R&D investment or it can be bought in the market (e.g. new machinery or patent licensing). If we study the effects of employment protection we should therefore take into account that firms are able to restore their productivity.

The academic literature has already documented different effects of EPL, which seem contradictory at first sight. One strand of the literature documented a negative effect of employment protection on productivity through inefficient worker reallocation.\footnote{Negative productivity effects from inefficient labor reallocation are found by Hopenhayn and Rogerson (1993), Griliches and Regev (1995), Olley and Pakes (1996), Foster, Haltiwanger, and Krizan (2001), Disney, Haskel, and Heden (2003), Baldwin and Gu (2006), Autor, Kerr, and Kugler (2007), and Bartelsman, Haltiwanger, and Scarpetta (2009) among others. Pierre and Scarpetta (2004) report that EPL particularly harms the growth prospects of medium sized firms.} Another strand of the literature has shown that EPL increases incentives to innovate and train. Pierre and Scarpetta (2004), for example, also shows that EPL incentivize firms to invest more in training. Acharya, Baghai, and Subramanian (2014) exploit the staged adoption of wrongful discharge laws in the U.S. to show that EPL spurs innovation and new firm creation, and Koeniger (2005) shows that countries with strict EPL tend to specialize in improving existing products.\footnote{Other authors also emphasize different positive aspects of EPL: Bertola (1994) shows that despite EPL lowers returns to irreversible investment and thus the speed of capital accumulation, it shifts the income distribution towards workers with no capital income. This explains why trade unions often favor stricter EPL. Kessing (2006) argues that firms facing EPL have a stronger average market position as they can credibly commit to fiercely defend their position against potential competitors, because EPL makes market exit very costly.} Theoretically, the positive effect on training and innovation can be explained by the decreased fluctuation of employees (see Zoega and Booth (2003) and Wasmer (2006)), the increased cost of laying off innovating and thus sometimes under-performing employees (see Acharya, Baghai, and Subramanian (2014)), and firms’ interest to improve existing products in order to ensure their competitiveness (see Koeniger (2005)). All these explanations are important to understand how EPL affects the economy. However, since they focus on how firms adjust their organization, they are not able to explain why the innovation and manufacturing sectors grow (see Acharya,
Baghai, and Subramanian (2014) and Autor, Kerr, and Kugler (2007)) at the expense of others (Autor, Donohue, and Schwab (2006)).

Our paper explains all these effects by employing an equilibrium matching model with an imperfect labor and innovation market to provide a novel explanation for why EPL lowers productivity while potentially boosting innovation. If employment protection is introduced, it becomes too expensive to lay off workers. This is the reason why firms’ willingness to pay for product or process innovations increases in order to bring their workforce back into production sooner. This shifts economic activity towards firms specializing in process and product innovation and triggers entry of new start-ups. By allowing for both the negative misallocation and the positive innovation effect we are also able to show (in the calibration part) that the positive innovation effect is only a second-best response to the introduction of EPL.\(^3\)

We assume labor market frictions, because without labor market frictions laid off workers could be reemployed immediately by other firms, which makes employment protection redundant. We assume frictions in the innovation market, because without frictions firms could immediately purchase the machinery (process innovation) or product idea (product innovation) necessary to restore productivity. We model both markets as matching markets, where the time to find an appropriate trading partner depends on the ratio of buyers to sellers in the market, and where prices are negotiated bilaterally. The interaction between labor and innovation market has the following implication. Employment protection induces firms to keep workers employed even if productivity has dropped. This increases firms’ willingness to pay for product or process innovations in order to restore productivity. This increases the price for innovations, triggers entry of new start-ups and shifts economic activity towards firms specializing in process and product innovation. It hence increases the rate at which firms, which are hit by a negative productivity shock, can purchase the (process or product) innovation necessary to restore their productivity. However, the positive innovation effect is not strong enough to dominate the negative misallocation effect of workers caused by EPL.

We calibrate our model to match aggregate US labor and product market statistics as well as aggregate firm exit and entry rates. We then take the calibrated model, introduce employment protection and show that the rate at which firms are able to restore their productivity increases.

\(^3\)Our model considers a steady state and does not model economic growth explicitly. We consequently can only show a sectorial shift towards more innovation induced by EPL. Whether this shift of resources towards the innovation sector leads finally to higher growth is unclear.
Our comparative static results are also in line with the estimated negative impact of wrongful dismissal laws on productivity found by Autor, Kerr, and Kugler (2007) and the positive effect on innovations shown by Acharya, Baghai, and Subramanian (2014). Both exploit the fact that from 1970 to 1999 13 US states introduced wrongful dismissal laws by recognizing the so-called "good-faith" exception to the employment-at-will doctrine. Our calibration results are also consistent with the findings by Acharya, Baghai, and Subramanian (2014), who show that the adoption of wrongful dismissal laws increases the number of firms, especially start-ups. We also find evidence for a shift in economic activity. More precisely we find that the number of firms producing the final consumption good decreases while the number of firms specializing in producing machinery (process innovation) or product ideas (product innovation) increases. These results can reconcile the findings by Autor, Kerr, and Kugler (2007), who observe an increase in employment in the manufacturing sector, with the findings by Autor, Donohue, and Schwab (2006), who find a negative effect on state-level employment.

The papers that are most closely related to ours are Wasmer (2006) and Bartelsman, Gautier, and De Wind (2016). Both papers investigate the effect of employment protection in an equilibrium matching model to explain differences between the United States and continental Europe. Wasmer (2006) investigates the effect employment protection has on the type of human capital investment undertaken in the economy. The main difference to our framework is that he models productivity shocks as exogenous, while we endogenize the rate at which firms are able to restore their productivity. Bartelsman, Gautier, and De Wind (2016) consider an equilibrium matching model where, under employment protection, firms are less likely to adopt a high-risk and high-return technology and more likely to adopt a low-risk and save technology. The main difference to our model is that they do not consider that employment protection can increase the returns to investment in innovation.

Section 2 is the theory part of this paper, where we discuss key equations of our model. The details of the model are deferred to the Appendix. In the calibration in Sections 3 we discuss the effects of the introduction of employment protection first without and then with the channel of the innovation market. Section 4 concludes.
2. Theory

2.1. Framework

The model has an infinite horizon, is set in continuous time and concentrates on steady states. All agents are risk neutral and discount the future at rate $r$. The economy is populated by a unit mass of homogenous workers and an endogenous mass $m$ of firms.

The production function of a firm $i$ for the consumption good is given by $y_i F(N_i) = y_i N_i^\alpha$, where $N_i \in R_0^+$ denotes the labor input chosen by firm $i$, and $y_i$ can be interpreted as the machinery, which the firm employs, or the profitability of the firm’s product in the market. The input factor $y_i$ takes the fixed value $y$ as long as the machinery is working or the product idea is profitable. Once a productivity shock hits, which happens at the exogenous rate $\delta$, the machine or the demand for a product breaks down the factor $y_i$ takes the value 0.\footnote{We use this binary productivity distribution in order to avoid the complications arising from a continuum of firm sizes, wages and innovation prices} Thus, $1/\delta$ can be interpreted as a product’s or machinery’s life-cycle. The input factor $y$ can be produced by each firm at its firm-specific innovation cost $k_i$. The research process underlying the production of the input $y$ is stochastic and happens at the Poisson rate $\eta$. We will also refer to the input factor $y$ as an innovation. It can be thought of both, a process innovation (machinery) or a product innovation. The per period cost $k_i$ is drawn randomly from a distribution characterized by the pdf $\xi(k)$ and the cdf $\Xi(k)$ on the support $[0, k_{\text{max}}]$.

Firms choose to become one of the following types $t \in \{S, B, R\}$ depending on the firm-specific innovation costs $k_i$. Type $S$ (seller) firms develop product ideas or produce capital goods (machinery), i.e., they produce the input $y$ at rate $\eta$. Once they have produced the input $y$, they will sell it on the innovation market. For simplicity, we assume that type $S$ firms cannot produce $y$ while they are busy with selling the input $y$ in the innovation market. Type $B$ (buyer) and type $R$ (researcher) firms produce the final consumption good as long as they are productive ($y_i = y$). All final consumption good producers sell the same homogenous good with prices normalized to unity. Type $B$ firms, which have been hit by a productivity shock ($y_i = 0$), search the innovation market for a new product or process innovation to restore their productivity to $y$. The details of the innovation market are given below. Type $R$ firms, which are hit by a productivity shock ($y_i = 0$), do their own research to restore their productivity to $y$. For simplicity we assume that firms cannot innovate while producing consumption goods.
The innovation market or market for new product ideas is characterized by matching frictions, with a constant return to scale matching function that satisfies the usual Inada conditions. Tightness in the innovation market is defined as the ratio of firms looking for a new machine or a product idea \( (m^B) \) for the measure of type \( B \) firms) to the firms that specialize in innovation and sell the input good \( y \) on the innovation market \( (m^S) \) for the measure of type \( S \) firms), i.e., \( \varphi = m^B / m^S \). Type \( S \) firms that sell the innovation \( y \) are matched at rate \( \varphi g (\varphi) \) with buyers (type \( B \) firms) and type \( B \) firms contact sellers (type \( S \) firms) at rate \( g (\varphi) \). The properties of the matching function are such that the matching probability of a seller (buyer) increases (decreases) with the ratio of buyers to sellers, i.e., \( [\varphi g (\varphi)]' > 0 \) and \( g' (\varphi) < 0 \). The innovation price is determined by Nash-bargaining where \( \beta \) denotes the bargaining power of sellers.

Innovation or research costs are firm-specific and set at the beginning of a firm’s life. Formally, we assume that potential firms have to pay a cost \( F \) upon entry (sufficiently small to guarantee existence) in order to learn the per-period, firm-specific innovation cost \( k_i \). For simplicity, we assume that new firms are born with input \( y_i = y \) upon paying the entry cost \( F \).

There are two types of shocks, a productivity or taste shock \( \delta \), which renders the machine or the product idea unprofitable (transforming \( y_i \) from \( y \) to \( 0 \)), and a destruction (insolvency) shock \( \lambda_d \) for type \( B \) and \( R \) firms and \( \lambda_s \) for type \( S \) firms, which forces the respective firm to close down because of insolvency.\(^5\) We assume that insolvency hits at an exogenous rate and abstract from an endogenous insolvency decision. Since an insolvency shock makes only sense, if the machine or the product idea is no longer competitive, we assume that a destruction shock can only occur, if a productivity shock has hit \( (y_i = 0) \). We assume that firms, which are hit by a productivity shock, are not insolvent and thus have the legal obligation and the financial resources to pay the firing cost \( f \) per laid off worker. This is no longer the case once the firm has to close down because of insolvency, since employment protection legislation usually regards insolvency as a just reason to lay off workers. We therefore assume that no firing costs are paid in case a destruction (insolvency) shock hits.

In order to ensure a similar life expectancy for type \( S \) and type \( B \) and \( R \) firms, we have to account for the fact that type \( S \) firms are in the "\( y_i = 0 \)"-state not only if a productivity shock \( \delta \) occurs (like type \( B \) and \( R \) firms), but also, if they have sold the innovation on the market (at

\(^5\)We need two types of shocks in order to be able to model the shift in the industry structure towards more innovating firms (type \( S \) firms) due to EPL.
rate \( \varphi_g(\varphi) \). We therefore assume \( \lambda_s < \lambda_d \), in order to enable an equally long life expectancy of all firms.

The labor market for production workers is also modeled using matching frictions. Firms hire workers by posting vacancies at the per period cost \( c \) (sufficiently small to guarantee existence). The matching function for production workers has constant return to scale and satisfies the Inada conditions. Labor market tightness is denoted by \( \theta = V/U \), where \( V \) equals the number of vacancies created by all firms and \( U \) the number of unemployed workers. The job finding rate of workers is given by \( \theta \lambda_m(\theta) \) and the rate at which firms contact workers by \( \lambda_m(\theta) \). The properties of the matching function are such that the matching probability of an unemployed worker (vacancy) increases (decreases) with the ratio of vacancies to unemployed, i.e., \( [\theta \lambda_m(\theta)]' > 0 \) and \( \lambda_m(\theta) < 0 \). Wages are negotiated and renegotiated each time the productivity of a firm changes. The bargaining power of workers is denoted by \( \gamma \). Unemployed workers receive unemployment benefits \( z \). Employed workers receive a wage \( w^{t,t}(y_i,N_i) \), which depends on \( y_i \in \{0,y\} \), on the marginal product \( y_iF'(N_i) \), and the type \( t \) of the firm. The wage will also depend on whether a worker is an insider (indexed by \( l = I \)), who is protected by employment protection, or an outsider (indexed by \( l = O \)), who is not. Newly hired workers are outsiders and worker, who stay employed after a productivity shock hits (and after productivity is restored) are insiders.

2.2. Optimality conditions and equilibrium

**Consumption good producers:** Firms of the consumption goods sector, i.e., type \( t \in \{B,R\} \) firms, choose their labor input by deciding on the number of vacancies \( V^t \) they want to post and the number of workers they want to lay off \( L^t \). The equation governing the change in the number of workers employed at firm \( i \) that posts vacancies \( V^t_i \) and lays off \( L^t_i \) workers is given by,

\[
\dot{N}^t_i = \lambda_m(\theta) V^t_i - L^t_i. \tag{1}
\]

Type \( B \) or \( R \) firms will post vacancies subject to equation (1) until the marginal value of a newly hired worker equals the expected cost of hiring a worker, i.e.,

\[
\frac{\partial \pi^t(N^t_i,L^t_i,y,k_i)}{\partial N^t_i} = \frac{c}{\lambda_m(\theta)}. \tag{2}
\]

New firms, which want to start production, will immediately hire their optimal number of workers \( N^t_i \), by posting \( V^t_i = N^t_i/\lambda_m(\theta) \) vacancies. If the marginal value of a newly hired worker
for a type B firm is different than that of a type R firm, then the number of vacancies posted and the number of workers employed will be different too.

Let us now consider firms, which have been hit by a productivity shock, i.e., \( y_i = 0 \), and laid off \( L_i^t \) workers. They pay the respective wage \( w^{I,R} \left( 0, N_i^R - L_i^R \right) \) for the \( N_i^R - L_i^R \) insiders that have not been laid off. To restore productivity type R firms will do own research, which costs \( k_i \) per period. At rate \( \eta \) they are successful in restoring their productivity. Once they have restored their productivity, they decide on how many workers \( H_i \) to hire in order to continue production.

If they are hit by the destruction shock \( \lambda_d \) they become insolvent, have to close down, and to lay off all their workers. The continuation value is zero. The Bellman equation for type B firms is only different in the way these firms restore their productivity. They buy it on the innovation market, where they meet sellers at rate \( g(\varphi) \) and pay the price \( p(k_j, N_i^B - L_i^B) \). The respective Bellman equations for type R and B firms while their productivity is zero \((y_i = 0)\) are given by,

\[
\begin{align*}
(r + \lambda_d) \pi^R \left( N_i^R, L_i^R, 0, k_i \right) &= -w^{I,R} \left( 0, N_i^R - L_i^R \right) \left( N_i^R - L_i^R \right) - k_i \\
&\quad + \eta \left( \max_H \left[ \pi^R \left( N_i^R - L_i^R + H, L_i^R, y, k_i \right) - \frac{c}{\lambda_m(\theta)} H \right] - \pi^R \left( N_i^R, L_i^R, 0, k_i \right) \right),
\end{align*}
\]

and

\[
\begin{align*}
(r + \lambda_d) \pi^B \left( N_i^B, L_i^B, 0, k_i \right) &= -w^{I,B} \left( 0, N_i^B - L_i^B \right) \left( N_i^B - L_i^B \right) \\
&\quad + g(\varphi) \int_0^{k_{\text{max}}} \max \left[ S^B, 0 \right] h(k_j) dk_j,
\end{align*}
\]

where the surplus from buying an process or product innovation \( S^B \) in the innovation market is given by,

\[
S^B = \max_H \left[ \pi^B \left( N_i^B - L_i^B + H, L_i^B, y, k_i \right) - \frac{c}{\lambda_m(\theta)} H \right] - \pi^B \left( N_i^B, L_i^B, 0, k_i \right) - p(k_j, N_i^B - L_i^B).
\]

By \( h(k_j) \) we denote the pdf of those firms that are willing to sell their innovations and have innovation cost \( k_j \).\(^6\)

Since the marginal value of a newly hired worker is the same irrespective of whether the firm just entered the labor market or restored its productivity after a productivity shock \( \delta \), i.e.,

\[
\frac{\partial \pi^I \left( N_i^I, L_i^I, y, k_i \right)}{\partial N_i^I} = \frac{\partial \pi^I \left( N_i^I - L_i^I + H, L_i^I, y, k_i \right)}{\partial H},
\]

\(^6\)In equilibrium \( h(k_j) = \xi(k_j) / \Xi(k^*) \), since all firms with \( k_j \) below some threshold \( k^* \) prefer to specialize in innovation and are willing to sell their innovations.
firms, which have laid off $L_i^t$ workers, will, once they have restored their productivity, hire $L_i^t$ workers to restore their optimal employment level $N_i^t$ by posting $V_i^t = L_i^t/\lambda_m(\theta)$ vacancies.

Hence at both types’ of firms there will be $N_i^t - L_i^t$ insiders and $L_i^t$ outsiders once the firm (re)starts production. This gives the following Bellman equation,

$$r \pi^t (N_i^t, L_i^t, y, k_i) = y (N_i^t)^\alpha - w^I^t (y, N_i^t) (N_i^t - L_i^t) - w^O^t (y, N_i^t) L_i^t$$

$$+ \delta \left( \max_L \left[ \pi^t (N_i^t, L, 0, k_i) - fL \right] - \pi^t (N_i^t, L_i^t, y, k_i) \right),$$

for $t \in \{B, R\}$.\(^7\) If a productivity shock $\delta$ hits, a firm has to decide on the number of workers it wants to lay off. In Appendix Appendix A we show that type $R$ and $B$ firms lay off all or none workers, since the decision to lay off a worker is independent of the number of workers laid off (or employed). The respective layoff decisions are given by,

$$- \frac{\eta \left( - \frac{w^{I^t,R} (0, \cdot)}{\eta} + \frac{c}{\lambda_m(\theta)} - \gamma f \right)}{r + \lambda_d + \eta} > f,$$

$$- \frac{g(\varphi) \left( - \frac{w^{I^t,B} (0, \cdot)}{g(\varphi)} + (1 - \beta) \left( \frac{c}{\lambda_m(\theta)} - \gamma f \right) \right)}{r + \lambda_d + g(\varphi)(1 - \beta)} > f,$$

for type $R$ and $B$ firms, respectively. Note, that the wages $w^{I^t,R} (0, \cdot)$ and $w^{I^t,B} (0, \cdot)$ are independent of the number of workers employed (see Appendix Appendix B). The reason is that the marginal product is zero and recruiting costs linear in $N_i^t$. The exact wage expressions are derived in Appendix Appendix B. The intuition behind the layoff decisions (4) and (5) are as follows. In the absence of a product idea workers are not productive, i.e., the marginal product of labor is zero. If a firm keeps a worker it has to pay the respective wage over the expected duration until the firm obtains a new innovation, i.e., $1/\eta$ or $1/g(\varphi)$. On the other hand, the firm saves the hiring cost, $c/\lambda_m(\theta)$, minus the part of the firing cost that the firm would have to bear, $\gamma f$, if it lays off a worker (the fraction $(1 - \beta)$ for type $B$ firms due to Nash-bargaining over innovation prices). These effects (left hand side) determine the marginal value of an additional worker if the worker is kept despite the fact that productivity is zero. If the marginal value of an additional worker is higher than the firing cost, then firms keep their workers. Otherwise

\(^7\)If a productivity shock $\delta$ hits, it is only optimal for the firm to close down, if the innovation market breaks down. Since the focus of the paper is to analyze the effect of EPL in the face of an operating innovation market we abstract from this case.
they lay off all their workers. The respective Bellman equations for $L^t_i = N^t_i$ and $L^t_i = 0$ can be found in Appendix A.

Wages in the labor market are determined by Nash-bargaining. We assume intra-firm bargaining as in Smith (1999), Cahuc and Wasmer (2001), and Cahuc, Marque, and Wasmer (2008), among others. The worker surplus equals the value of being employed minus the outside option of being unemployed. The firm’s surplus depends on whether it bargains with outsiders (new workers) or with insiders. If a firm is bargaining with outsiders the surplus is given by the marginal value of an additional worker. If an old firm is renegotiating the wages of its current workforce (insiders), then the surplus of continuing the employment relationship is given by the marginal value of an additional worker plus the firing cost $f$, since a bargaining agreement ensures that the firm does not have to pay the firing cost. The Nash-product in the event a firm negotiates with outsiders and insiders, respectively, is given by,

$$w^{O,t} (y_i, N^t_i) = \arg \max_w \left( W^{O,t} (w) - U \right)^\gamma \left( \frac{\partial \pi^{O,t} (N^t_i, y_i, k_i)}{\partial N^t_i} \right)^{1-\gamma},$$

$$w^{I,t} (y_i, N^t_i) = \arg \max_w \left( W^{I,t} (w) - U \right)^\gamma \left( \frac{\partial \pi^{I,t} (N^t_i, y_i, k_i)}{\partial N^t_i} + f \right)^{1-\gamma}.$$ 

**Innovation producers:** Firms of the innovation sector, i.e., type $S$ firms, specialize on producing and selling innovations, i.e., producing and selling the input $y$ for consumption good produces. The expected discounted profit of a type $S$ firm $\pi^S (0, y_j, k_j)$ depends on the prices $p (k_j, N^B_i)$ it receives for its innovation. Prices are determined by Nash-bargaining, i.e.,

$$p (k_j, 0) = \arg \max_p \left( \pi^{O,B} (N^B_i, y, k_i) - \frac{c}{m (\theta)} N^B_i - \pi^{O,B} (0, 0, k_i) - p \right)^{1-\beta} \times \left( p + \pi^S (0, 0, k_j) - \pi^S (0, y, k_j) \right)^\beta,$$

$$p (k_j, N^B_i) = \arg \max_p \left( \pi^{I,B} (N^B_i, y, k_i) - \pi^{I,B} (N^B_i, 0, k_i) - p \right)^{1-\beta} \times \left( p + \pi^S (0, 0, k_j) - \pi^S (0, y, k_j) \right)^\beta,$$

The price $p (k_j, N^B_i)$ of the innovation will depend on the surplus that is generated by the innovation. The surplus will depend on the type $S$ firm’s own innovation cost $k_j$ and on the number of workers employed at the buyer $N^B_i$. The surplus of a type $B$ firm that buys an innovation is given by the increase in expected profits from restoring productivity $y_i$ from 0 to
y, which allows the firm either to bring its workforce back to productive use, if it has kept its workforce after the productivity shock, or to hire and productively employ new workers, if it has laid off its workforce. The buyer’s innovation cost \( k_i \) does not enter the surplus, since a firm that decided to buy the innovation \( y \) will also do so in the future, that is, it will never decide to do own research. The surplus of a type \( S \) firm that sells the innovation is given by the price plus the expected loss in profit \( \pi^S(0,0,k_j) - \pi^S(0,y,k_j) \) from having to produce a new innovation. A type \( S \) firm will not be active on the labor market for production workers, i.e., \( N_j^S = 0 \), since innovation requires by assumption no production workers. Labor market conditions only enter a type \( S \) firm’s expected discounted profit \( \pi^S(0,y,k_j) \) via the number of workers employed by type \( B \) firms, which influences the price \( p(k_j,N_i^B) \).

**Specialization:** We can now characterize which firms will enter the consumption goods sector and specialize on production of the final good without doing own research, type \( B \) firms, which firms will enter the innovation goods sector and specialize on innovation, type \( S \) firms, and which firms will do both produce consumption goods and do own research, type \( R \) firms. Given the innovation cost \( k_i \) each firm will choose its type \( t \) such that expected profits are maximized, i.e.,

\[
\max_{t \in \{S,B,R\}} \pi^{O,t}(N_i^t,y,k_i) - \frac{c}{\lambda_m(\theta)} N_i^t,
\]

where \( N_i^t \) denotes the optimal number of workers that the firm intends to hire following the optimal vacancy creation condition in equation (2).

Type \( B \) firms decide to buy an innovation when they are hit by a productivity shock. They therefore never innovate. Their expected profits hence are independent of \( k_i \). Thus, the minimum profit that each firm can obtain is given by the expected profit of type \( B \) firms. Type \( R \) firms do their own research, if they are hit by a productivity shock. Type \( S \) firms specialize in innovation and do more research than type \( R \) firms. Their profits are therefore more sensitive to the cost of innovation \( k_i \). In Appendix Appendix E we formally show that the expected profit of type \( S \) firms decreases more in the cost of innovation \( k_i \) than the expected profit of type \( R \) firms, i.e.,

\[
\frac{\partial \pi^S(0,y,k_i)}{\partial k_i} < \frac{\partial \pi^{O,R}(N_i^R,y,k_i)}{\partial k_i} < \frac{\partial \pi^{O,B}(N_i^B,y,k_i)}{\partial k_i} = 0.
\]

This is also shown in Figure 1.

Given this single crossing property we can define the innovation cost thresholds \( k^* \) and \( k^{**} \). Firms with innovation cost \( k_i \in [0,k^*] \) will specialize in innovation, firms with innovation cost
Figure 1 – Specialization decision of firms

$k_i \in (k^*, k^{**})$ will produce consumption goods and do their own research if they are hit by a productivity shock, and firms with innovation cost $k_i \in [k^{**}, k_{\text{max}}]$ will produce consumption goods and buy a new innovation when they need one. The thresholds are formally defined by the following indifference conditions for type $S$ and type $R$ firms (thresholds $k^*$) and type $R$ and type $B$ firm (thresholds $k^{**}$) respectively,

$$\pi^S(0, y, k^*) = \pi^{O,R}(N_i^R, y, k^*) - \frac{c}{\lambda_m(\theta)} N_i^R,$$

$$\pi^{O,R}(N_i^R, y, k^{**}) - \frac{c}{\lambda_m(\theta)} N_i^R = \pi^{O,B}(N_i^B, y, k^{**}) - \frac{c}{\lambda_m(\theta)} N_i^B.$$  \hfill (8)

Remember that the appropriate equations for the expected profits depends on whether firms lay off workers if a productivity shock hits.

**Firm entry:** The expected profit of a new firm before it draws its innovation cost $k_i$ determines the number of active firms $m$ in the economy. Since the expected profits $\pi^S(0, y, k_i)$ and $\pi^{O,R}(N_i^R, y, k_i)$ are linear in $k_i$ and $\pi^{O,B}(N_i^B, y, k_i)$ independent of $k_i$, we can write the free
entry condition as,

$$F = \Xi (k^*) \pi^S (0, y, \overline{k})$$

$$+ (\Xi (k^{**}) - \Xi (k^*)) \left( \pi^O, R \left( N^{R_i}, y, \overline{k} \right) - \frac{c}{\lambda_m (\theta)} N^{R_i} \right)$$

$$+ (1 - \Xi (k^{**})) \left( \pi^O, B \left( N^{B_i}, y, k_i \right) - \frac{c}{\lambda_m (\theta)} N^{B_i} \right),$$

where average innovation cost \( \overline{k} \) among type \( S \) firms and \( \overline{k} \) among type \( R \) firms are given by,

$$\overline{k} = \int_0^{k^*} k_i \frac{\xi (k_i)}{\Xi (k^*)} dk_i$$

and

$$\overline{k} = \int_{k^*}^{k^{**}} k_i \frac{\xi (k_i)}{\Xi (k^{**}) - \Xi (k^*)} dk_i.$$ 

Given the entry cost \( F \), firms will enter until the expected profit is equal to the cost of entry. The parameter \( m \) for the number of firms is not directly visible in the entry condition (10), but it enters the expected profit indirectly via the labor market tightness \( \theta \). The steady state value of the labor market tightness is determined using the steady state flow equations analyzed next.

**Steady state flows:** We denote the measure of unemployed workers by \( u \) and the measure of type \( t \) firms with \( N^t_i \) employed workers and with productivity \( y_i \in \{0, y\} \) by \( m^t \left( y_i, N^t_i \right) \), where the number of firms must sum up to \( m \). The respective worker- or firm-level flow measures evolve according to the difference between in- and outflows as shown in Appendix Appendix E.1. We focus on the steady state.

Steady state unemployment is given by,

$$u = \begin{cases} 
\frac{\lambda_d}{\theta \lambda_m (\theta)} \left( m^B \left( 0, N^{B_i} \right) N^{B_i} + m^R \left( 0, N^{R_i} \right) N^{R_i} \right) & \text{if } L^I_i = 0, \\
\frac{\delta}{\theta \lambda_m (\theta)} \left( m^B \left( y, N^{B_i} \right) N^{B_i} + m^R \left( y, N^{R_i} \right) N^{R_i} \right) & \text{if } L^I_i = N^t_i.
\end{cases}$$

If all \( R \) and \( B \) type firms retain their workers if they are hit by a productivity shock, the inflow into unemployment is given by the rate \( \lambda_d \) at which consumption good producers are forced to lay off workers due to a destruction shock times the number of workers that are employed at these firms, i.e., \( m^B \left( 0, N^{B_i} \right) N^{B_i} + m^R \left( 0, N^{R_i} \right) N^{R_i} \). If all firms lay off their workers if they are hit by a productivity shock, the inflow into unemployment is given by the rate \( \delta \) at which a productivity shock hits times the number of workers employed at firms producing consumption goods, i.e., \( m^B \left( y, N^{B_i} \right) N^{B_i} + m^R \left( y, N^{R_i} \right) N^{R_i} \). The outflow is given by the matching probability of unemployed workers times the number of unemployed workers \( \theta \lambda_m (\theta) u \).
The firm-level flow equations (see Appendix Appendix E.1) allow us to write the ratio of the steady state measures of type B firms \( m^B (0, N^B_i) \) to the measure of type S firms \( m^S (y, 0) \), which determines the ratio \( \varphi \) of buyers to sellers in the innovation market and hence the meeting probability of buyers and sellers, i.e.,

\[
\varphi = \frac{m^B (0, N^B_i)}{m^S (y, 0)} = \frac{\lambda_s}{\lambda_s + \eta} \frac{\delta + \varphi g (\varphi) 1 - \Xi (k^{**})}{\lambda_d \Xi (k^*)}.
\]

This equation implicitly determines the innovation market tightness \( \varphi \). The innovation market tightness \( \varphi \) decreases with both innovation cost thresholds \( k^* \) and \( k^{**} \), since, in the case of \( k^* \), more firms decide to specialize in innovation and, in case of \( k^{**} \), fewer firms decide to buy a new innovation when they are hit by a productivity shock.

**Equilibrium:** The equilibrium in this economy is characterized by the market tightness in the innovation market \( \varphi \) and the labor market \( \theta \), the layoff decision of type B and R firms \( L^B_i \) and \( L^R_i \), the threshold values \( k^* \) and \( k^{**} \) of the innovation cost \( k_i \) that determine the fraction of type S, B, and R firms and the number of active firms in the economy \( m \), i.e., by the set of variables \( \{ \varphi, \theta, L^B_i, L^R_i, k^*, k^{**}, m \} \). We concentrate on an equilibrium in which all three types exist. Of course there are parameter values where only S and B type firms exist (for \( \eta \) sufficiently small), and parameter values where only type R firms exist (for \( \eta \) sufficiently high). In Appendix Appendix F we shows that the equilibrium is unique and can be solved sequentially.

### 3. Calibration

In this section we show that our model is able to reconcile the empirical findings that the introduction of wrongful dismissal laws in the US lead to a decrease in productivity as shown by Autor, Kerr, and Kugler (2007) and an increase in the number of active firms and the number of patents as shown by Acharya, Baghai, and Subramanian (2014).

#### 3.1. Baseline calibration

**Parameters and targets:** The model comprises of 17 exogenous parameters (see Table 1). In the calibration we choose the time period to represent one quarter and set the quarterly discount rate to \( r = 0.012 \) (equivalent to an annual discount factor of 0.953).

The parameters to target aggregate labor market statistics are taken from Shimer (2005) and Kaas and Kircher (2015) among others. We use a standard Cobb-Douglas type matching function, i.e., \( M(U, V) = \kappa_i U^\psi V^{1-\psi} \). Like Shimer (2005) we target a job finding rate \( \theta \lambda_m (\theta) \)
Moreover, we target an unemployment rate in line with the long run US average (4.5% to 5%). To do so we set the labor market matching efficiency parameter to $\kappa_l = 2$ and the vacancy posting costs to $c = 0.0352$. The matching elasticity on the labor market $\psi$ is set at a medium value of 0.5. As workers in our model are all production workers unemployment benefits are set at a fairly high value $z = 0.575$, implying a replacement rate of 85%, which is close to Hagedorn and Manovskii (2008). Finally, workers’ bargaining power $\gamma$ is set at 0.72 (see Shimer (2005)). To specify the parameters of the production function for large firms we follow Kaas and Kircher (2015). We normalize the productivity parameter to $y = 1$ and set the labor elasticity parameter of the production function $\alpha$ equal to the labor share of 0.7. Bauer and Lingens (2014), who also calibrate a matching model with large firms, take a value of 0.8 for the labor elasticity parameter. They motivate their choice by targeting realistic mark-up values. Taking a value of 0.8 instead of 0.7 for labor elasticity would change our results quantitatively but not qualitatively.

### Table 1 – Exogenous parameters values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source / Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.100</td>
<td>Target: Average product cycle length, see Magnier, Kalaitzandonakes, and Miller (2010).</td>
</tr>
<tr>
<td>$\lambda_d$</td>
<td>0.250</td>
<td>Target: Average firm life expectancy of 50 quarters, see Burns (2010).</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>0.010</td>
<td>Set to equal $25\lambda_d = \lambda_s$.</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.356</td>
<td>Set to equal the ratio of average product life cycle length to time to market of 3.56, see Griffin (2002).</td>
</tr>
<tr>
<td>$y$</td>
<td>1.000</td>
<td>Normalization.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.700</td>
<td>Set to equal the labor share, see Kaas and Kircher (2015).</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.500</td>
<td>Set to the medium value, see Mortensen and Pissarides (1994).</td>
</tr>
<tr>
<td>$\kappa_l$</td>
<td>2.000</td>
<td>Target: Average job-finding rate of 1.36 (Shimer (2005)) and unemployment rate of about 5%.</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.500</td>
<td>Set due to computational constraints.</td>
</tr>
<tr>
<td>$\kappa_p$</td>
<td>0.180</td>
<td>Set to get a product idea finding rate of $g(\varphi) = \eta$.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.720</td>
<td>Set to an conventional value Shimer (2005).</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.500</td>
<td>Set to equal the elasticity of the innovation market matching function.</td>
</tr>
<tr>
<td>$z$</td>
<td>0.575</td>
<td>Target: Replacement rate of 85%.</td>
</tr>
<tr>
<td>$c$</td>
<td>0.035</td>
<td>Target: Average job-finding rate of 1.36 (Shimer (2005)) and unemployment rate of about 5%.</td>
</tr>
<tr>
<td>$r$</td>
<td>0.012</td>
<td>Compare Shimer (2005).</td>
</tr>
<tr>
<td>$f$</td>
<td>1.000</td>
<td>Set to equal 4.5 months of production, see Bartelsman, Gautier, and De Wind (2016).</td>
</tr>
<tr>
<td>$F$</td>
<td>2.88</td>
<td>Set to get an average of 2.58 production workers per establishment (see U.S. Census (2007)).</td>
</tr>
</tbody>
</table>

We assume that research costs are uniformly distributed between zero and one. The support of the research cost distribution is chosen such that the threshold values for the investment cost can be directly used to obtain the shares of the respective firm types. Using the uniform
distribution on the [0, 1] support implies a R&D expenditure to GDP ratio of around 0.014, a value that is of the same magnitude as the 2% of GDP reported in Eurostat (2011) for private sector R&D expenditure in the US. The productivity shock rate $\delta$ is calibrated in order to reflect average product life-cycle length. Magnier, Kalaitzandonakes, and Miller (2010) find that on average products last for about 2.5 years, implying $\delta = 0.1$. In order to obtain a value for the research success rate $\eta$ we use a result by Griffin (2002), who finds that the ratio of product life cycle length to the time to market for the development of a new product is 3.56 in almost all industries (i.e., the product life cycle length and the time to market are extremely highly correlated across industries with $\rho = 0.99$). Given the ratio of product life cycle length to the time to market of 3.56 we set the research success rate at $\eta = 0.356$.

There is less information in the literature that we can use in order to pin down the parameters for the innovation market. We also use a Cobb-Douglas type matching functions for the innovation market, i.e., $P(S, B) = \kappa_p(S)^\nu B^{1-\nu}$. We set the exponent of the innovation market matching function to $\nu = 0.5$ in order to derive an explicit expression for the innovation market tightness, which is done to reduce the computer capacity necessary to solve the model numerically. The bargaining power of firms that sell their product ideas in the innovation market is also chosen to equal $\beta = 0.5$. We choose a matching efficiency in the innovation market of $\kappa_p = 0.18$ in order to obtain an innovation acquisition rate $g(\varphi)$ that is of roughly the same magnitude as the research success rate $\eta$ for firms that do their own research.

Firing costs $f = 1$ are chosen to equal 4.5 month of production in the calibration with employment protection and zero otherwise. Given the fact that only 13 US states have adopted the ”good-faith” exception, the value $f = 1$ seem appropriate since it implies roughly an average value of one month of production for the US as a whole.

The firm level destruction rates $\lambda_d$ and $\lambda_s$ are chosen such that the average life expectancy of firms lies around 50 quarters (see Burns (2010)). We set the destruction shock of producing firms to be much larger than the destruction shock of firms that specialize in innovation, i.e., $\lambda_d = 0.25$ and $\lambda_s = 0.01$, since type $S$ firms are more often exposed to the ”$y_i = 0$”-state then type $R$ and type $B$ firms given that $y_i = 0$ every time they sell their innovation.

Finally we set entry costs to $F = 2.88$, which leads to firm-level employment of 2.58 production workers at type $R$ and type $B$ firms. Since we do not include non-production workers we have chosen a value that is significantly smaller than the average US firm size of around 4.18
employees (production and non-production workers) documented by U.S. Census (2007).

**Baseline calibration of the US economy:** The first column of Table 2 below shows the baseline calibration of the US economy without employment protection. Given the normalization of the number of workers in the economy and the productivity parameter to one, final consumption output\(^8\) without employment protection is equal to 0.717. The total measure of innovations per quarter of 0.048 consist of the innovations done by existing firms 0.032 (innovations within), and firms, which enter the economy 0.016 (innovations upon entry).\(^9\) The private sector R&D expenditure to GDP ratio equals 0.014. Firms that acquire an innovation are willing to pay on average 1.027. The rate at which type B firms are able to acquire a new machinery or product idea \(y\) in the market for innovations equals 0.307. Taking the weighted average over type B and type R firms then the average duration in which a firm is in the low productivity state \(y_i = 0\) is slightly more than 9.2 months.

In steady state the free entry condition ensures that average expected profits exactly offset entry costs \(F\). This pins down the number of firms in the economy at \(m = 0.614\), out of which 0.369 produce the final consumption goods. The remaining firms either conduct own research 0.089 or search for a trading partner in the innovation market 0.155. The unemployment rate among production workers is given by 0.048.

### 3.2. Introducing employment protection

In order to shed light on the interaction of employment protection and innovations we first keep the innovation price fixed at the level without employment protection. Later, we endogenize the price to demonstrate the role of the innovation market.

**Fixed innovation price:** Table 2 compares the baseline model without employment protection with a situation in which employment protection is in place. However, the innovation price is

\[^8\]Final output of consumption goods is given by all type \(R\) and type \(B\) firms with productivity \(y_i = y\), i.e.,

\[Y = m^R \left(y, N_i^R\right)^\alpha + m^B \left(y, N_i^B\right)^\alpha + m^E \left(y, N_i^E\right)^\alpha.\]

\[^9\]In our model, there are two ways in which new innovations are created. All firms that enter the economy, i.e., \(m^e\), are assumed to start with an innovation. Additionally, research is done by all type \(S\) and type \(R\) firms with \(y_i = 0\). These firms produce a new innovation at the research success rate \(\eta\). The number of patents and patent citations in our framework is therefore measured by the number of innovations created each period, i.e.,

\[I = m^e + \eta \left(m^R(0, N_i^R) + m^S(0, 0)\right)\]
set at 1.027, the level in the baseline calibration. In order to understand the effect on profits, we first kept the number of firms in the economy constant at 0.614. This is shown in the second column of Table 2.

**Table 2 – Results: Employment protection with fixed idea price**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Baseline without EPL</th>
<th>With EPL m-fixed</th>
<th>With EPL m-flexible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final consumption output (Y)</td>
<td>0.717</td>
<td>0.635</td>
<td>0.618</td>
</tr>
<tr>
<td>Total innovations (I)</td>
<td>0.048</td>
<td>0.048</td>
<td>0.045</td>
</tr>
<tr>
<td>Total R&amp;D costs / GDP</td>
<td>0.014</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>Seller-researcher threshold (k*)</td>
<td>0.039</td>
<td>0.043</td>
<td>0.041</td>
</tr>
<tr>
<td>Researcher-buyer threshold (k***)</td>
<td>0.402</td>
<td>0.347</td>
<td>0.358</td>
</tr>
<tr>
<td>Innovation acquisition rate (g((\phi)))</td>
<td>0.307</td>
<td>0.309</td>
<td>0.302</td>
</tr>
<tr>
<td>Innovation price (p)</td>
<td>1.027</td>
<td>1.027</td>
<td>1.027</td>
</tr>
<tr>
<td>Unemployment rate (u)</td>
<td>0.048</td>
<td>0.044</td>
<td>0.059</td>
</tr>
<tr>
<td>Job finding rate ((\theta_{m}(\theta)))</td>
<td>1.998</td>
<td>0.803</td>
<td>0.600</td>
</tr>
<tr>
<td>Job destruction rate</td>
<td>0.100</td>
<td>0.037</td>
<td>0.037</td>
</tr>
<tr>
<td>Type R firms (N^R_i)</td>
<td>2.581</td>
<td>2.404</td>
<td>2.497</td>
</tr>
<tr>
<td>Type B firms (N^B_i)</td>
<td>2.581</td>
<td>2.217</td>
<td>2.290</td>
</tr>
<tr>
<td>Total number of firms (m)</td>
<td>0.614</td>
<td>0.614</td>
<td>0.574</td>
</tr>
<tr>
<td>Type S with (y_i = y)</td>
<td>0.115</td>
<td>0.125</td>
<td>0.112</td>
</tr>
<tr>
<td>Type S with (y_i = 0)</td>
<td>0.065</td>
<td>0.070</td>
<td>0.064</td>
</tr>
<tr>
<td>Type R with (y_i = y)</td>
<td>0.147</td>
<td>0.120</td>
<td>0.119</td>
</tr>
<tr>
<td>Type R with (y_i = 0)</td>
<td>0.024</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>Type B with (y_i = y)</td>
<td>0.222</td>
<td>0.237</td>
<td>0.220</td>
</tr>
<tr>
<td>Type B with (y_i = 0)</td>
<td>0.040</td>
<td>0.043</td>
<td>0.040</td>
</tr>
<tr>
<td>Firms with employment</td>
<td>0.369</td>
<td>0.419</td>
<td>0.398</td>
</tr>
<tr>
<td>Average firm destruction rate</td>
<td>0.027</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>Average profit</td>
<td>2.882</td>
<td>2.805</td>
<td>2.881</td>
</tr>
</tbody>
</table>

The introduction of employment protection implies that firms continue to employ their workers, if they are hit by a productivity shock. This increases the number of firms with employment by 14%. Although there are more firms, which employ workers, the number of firms producing the final consumption good decreases, because keeping and paying unproductive workers decreases profits, especially profits of type R and type B firms, and makes it more attractive to specialize in innovation. Higher labor costs also imply that firm-level employment drops on average by 11.6%. Both negative effects lead to a drop in final consumption output.

Unemployment falls (slightly) as job destruction decreases even more than job creation. The effect on job destruction emerges because under employment protection not only those type R and type B firms, which have not been hit by a productivity shock, but all type R and type B firms employ workers. The drop in the unemployment rate shown in the second column
does not yet take the negative effect of employment protection on firm entry and the respective (additional) negative effect on vacancy creation into account. The second column of Table 2 also shows that the share of type S firms slightly increases, as these firms are not affected by firing costs.

The adoption of employment protection laws decreases average profits by roughly 2.7% implying that the total number of firms in the economy with employment protection decreases by about 6.5%. This can be seen by looking at the third column of Table 2, which keeps the innovation prices constant, but allows for adjustment of the number of firms. The number of innovations also decreases with the number of firms by about 6.3%. Unemployment significantly increases (from 4.8% to 5.9%) once the additional effect of lower firm entry is taken into account.

Thus, without the innovation market channel (flexible innovation price) our model captures only the negative effects of worker misallocation due to EPL. By opening the innovation market channel we allow firms to react to the introduction of EPL by increasing their willingness to pay for innovations. This will enable us to explain the empirical findings by Acharya, Baghai, and Subramanian (2014), who find a positive effect of employment protection on the number of patents and an increase in the number of firms.

**Endogenous innovation price:** Until now we fixed the innovation price at its baseline value in order to disentangle the innovation market effect from the conventional profit depressing effects of employment protection. We now compare the baseline calibration with the model with employment protection under flexible innovation prices. Again, the second column of Table 3 keeps the number of firms in the economy at the baseline calibration level in order to understand the effects of employment protection on profits.

The introduction of employment protection increases labor cost during the period in which a firm keeps its workers although it has been hit by a productivity shock. This increases the willingness of firms, which have been hit by a productivity shock, to pay for an innovation. This leads to an increase in the innovation price from 1.027 in the baseline calibration to 1.365. This increases the profits of firms that specialize in innovation relative to the profit of final consumption good producers. The associated shift in the composition of firms increases the number of innovations by type S and type R firms, by around 10%. The total number of innovations does not change, however, since we keep the number of firms fixed, which implies
that we exclude all innovations that are attached to the entry of new firms.\footnote{As $\lambda_s \leq \lambda_d$ the change in the composition of firms towards more sellers leads to less exits per period and accordingly to less entries per period.}

The change in the composition of firms mainly increases the number of type $S$ firms that specialize in innovation. This increase is much larger compared to a situation with fixed innovation price (see Table 2) implying that the change in the innovation price is the main driver of the sectorial shift. At the same time the number of type $R$ and type $B$ firms that produce consumption goods decreases from 0.369 to 0.312. Although the reduction in the number of producing firms leads to lower hiring costs and higher profits, firm-level employment slightly decreases as firing costs effectively increase the marginal costs of employing a worker. Accordingly unemployment strongly increases (from 4.8\% to 7.9\%) whereas final consumption good production decreases by around 16.2\%.

**Table 3 – Results: Employment protection with endogenous idea price**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Baseline without EPL</th>
<th>With EPL m-fixed</th>
<th>With EPL m-flexible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final consumption output ($Y$)</td>
<td>0.717</td>
<td>0.601</td>
<td>0.632</td>
</tr>
<tr>
<td>Total innovations ($I$)</td>
<td>0.048</td>
<td>0.048</td>
<td>0.053</td>
</tr>
<tr>
<td>Total R&amp;D costs / GDP</td>
<td>0.014</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>Seller-researcher threshold ($k^*$)</td>
<td>0.039</td>
<td>0.064</td>
<td>0.064</td>
</tr>
<tr>
<td>Researcher-buyer threshold ($k^{**}$)</td>
<td>0.402</td>
<td>0.344</td>
<td>0.337</td>
</tr>
<tr>
<td>Innovation acquisition rate ($g(\varphi)$)</td>
<td>0.307</td>
<td>0.400</td>
<td>0.397</td>
</tr>
<tr>
<td>Innovation price ($p$)</td>
<td>1.027</td>
<td>1.365</td>
<td>1.323</td>
</tr>
<tr>
<td>Unemployment rate ($u$)</td>
<td>0.048</td>
<td>0.079</td>
<td>0.045</td>
</tr>
<tr>
<td>Job finding rate ($\theta \lambda_m(\theta)$)</td>
<td>1.998</td>
<td>0.396</td>
<td>0.722</td>
</tr>
<tr>
<td>Job destruction rate</td>
<td>0.100</td>
<td>0.034</td>
<td>0.034</td>
</tr>
<tr>
<td>Firm-level employment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type R firms ($N^R_i$)</td>
<td>2.581</td>
<td>2.568</td>
<td>2.444</td>
</tr>
<tr>
<td>Type B firms ($N^B_i$)</td>
<td>2.581</td>
<td>2.544</td>
<td>2.416</td>
</tr>
<tr>
<td>Total number of firms ($m$)</td>
<td>0.614</td>
<td>0.614</td>
<td>0.670</td>
</tr>
<tr>
<td>Type S with $y_i = y$</td>
<td>0.115</td>
<td>0.169</td>
<td>0.185</td>
</tr>
<tr>
<td>Type S with $y_i = 0$</td>
<td>0.065</td>
<td>0.084</td>
<td>0.092</td>
</tr>
<tr>
<td>Type R with $y_i = y$</td>
<td>0.147</td>
<td>0.088</td>
<td>0.095</td>
</tr>
<tr>
<td>Type R with $y_i = 0$</td>
<td>0.024</td>
<td>0.015</td>
<td>0.016</td>
</tr>
<tr>
<td>Type B with $y_i = y$</td>
<td>0.222</td>
<td>0.223</td>
<td>0.246</td>
</tr>
<tr>
<td>Type B with $y_i = 0$</td>
<td>0.040</td>
<td>0.034</td>
<td>0.038</td>
</tr>
<tr>
<td>Firms with employment</td>
<td>0.369</td>
<td>0.361</td>
<td>0.394</td>
</tr>
<tr>
<td>Average firm destruction rate</td>
<td>0.027</td>
<td>0.021</td>
<td>0.021</td>
</tr>
<tr>
<td>Average profit</td>
<td>2.882</td>
<td>2.996</td>
<td>2.882</td>
</tr>
</tbody>
</table>

In stark contrast to the calibration in Table 2 with fixed product idea prices, average profits increase by around 4\%. This triggers firm entry and increases in the number of firms in the new
steady state from 0.614 to 0.670. The increase in the number of firms of around 9.1% is well in line with the 8.7% to 12.4% increase estimated by Acharya, Baghai, and Subramanian (2014).

The increase in the total number of firms has a counteracting effect on the average innovation price, which decreases from 1.365 in the calibration with the fixed number of firms to 1.323. However, the above mentioned shift in the composition of firms towards a higher fraction of firms that specialize in innovation is still present and leads in combination with the innovations generated by newly created firms to an increase in total innovations of around 8.3%. This increase is slightly below the one estimated by Acharya, Baghai, and Subramanian (2014), which lies between 12.2% and 18.8%. This shift in economic activity towards firms that specialize in innovation also increases the innovation acquisition rate \( g(\varphi) \) at which type \( B \) firms can restore their productivity from 0.307 in the baseline calibration to 0.397. Taking the weighted average over type \( B \) and type \( R \) firms then the average duration in which a firm is in the low productivity state \( y_i = 0 \) is with employment protection equal to 7.8 months. This implies a decrease of 15.7% compared to the average duration without employment protection of 9.2 months.

The higher number of firms \( m \) and the higher innovation acquisition rate dampen the decrease in the number of firms producing consumption goods. Nevertheless it is still lower than in the calibration without employment protection (0.340 instead of 0.369). Together with the decrease in firm-level employment of around 6% on average, this still leads to a substantial decline in the production of final consumption goods by 11.8%. This shows that higher innovation activity in the economy is only a second best response to the introduction of EPL and cannot overcome the negative misallocation effect of labor.

Unemployment fully recovers from its high value in the calibration with fixed firm numbers once the increase in \( m \) is taken into account. Indeed, as unemployment falls to 4.5% it is even slightly below its original value, which was 4.8%. The model is therefore well in line with the empirical fact that employment protection can have ambiguous effects on unemployment. Since the decrease in final consumption output goes along with an increase in total employment, an increase in the number of firms, and an increase in the number of innovations our calibration is also able to explain the decrease in labor and total factor productivity due to employment protection observed by Autor, Kerr, and Kugler (2007).
4. Conclusion

We study the effects of employment protection taking into account that firms are able to restore their productivity. We develop an equilibrium matching model with an imperfect labor and innovation market. We model both markets as matching markets, where the time to find an appropriate trading partner depends on the ratio of buyers to sellers in the market, and where prices are negotiated bilaterally. The interaction between labor and innovation market has the following implication. Employment protection induces firms to keep workers employed even if productivity has dropped. This leads to a misallocation of labor. But it also increases firms’ willingness to pay for product or process innovations in order to restore productivity. This increases the price for innovations, triggers entry of new start-ups and shifts economic activity towards firms specializing in process and product innovation. It hence increases the rate at which firms that are hit by a negative productivity shock can purchase the (process or product) innovation necessary to restore their productivity.

We calibrate our model to match aggregate US labor and product market statistics as well as aggregate firm exit and entry rates. We then take the calibrated model, introduce employment protection and show that the rate at which firms are able to restore their productivity increases. Our comparative static results are also in line with the estimated negative impact of wrongful dismissal laws on productivity, the positive effect on innovations and the number of firms, especially start-ups. We also find evidence for a shift in economic activity towards firms specializing in producing machinery (process innovation) or product ideas (product innovation). The labor misallocation effect of EPL however dominates the innovation effect such that the production of the final consumption good decreases.

We therefore conclude that EPL is less harmful if an economy has the potential to innovate. Because only if the economic environment provides incentives to innovate, i.e., encourages start-ups (product innovation) and investments (process innovation), is it able to earn the fruits of the innovation enhancing effect of EPL.
References


Appendix

Appendix A. Value Functions

Workers: Workers can become employed only at firms that produce consumption goods, since innovation firms will not be active on the labor market for production workers. Firms will post vacancies and hire unemployed workers. Denote the fraction of vacancies posted by type \( t \in \{B, R\} \) firms with productivity \( y_i \) and \( N_i \) employed workers by \( v^t(y_i, N_i) \). We therefore can write the value of being unemployed as,

\[
r_U = z + \theta \lambda_m(\theta) \sum_{t \in \{B, R\}, y_i \in \{0, y\}} \max \left[ v^t(y_i, N_i) W^{O,t}(w^{O,t}(y_i, N_i)) - U, 0 \right], \tag{A.1}
\]

where the value of being employed at a type \( t \) firm as an outsider, i.e., as a newly hired worker (indexed by \( O \)), at the wage \( w^{O,t}(y_N) \) is given by,

\[
r^{W,O,t}(w^{O,t}(y_N)) = w^{O,t}(y_N) + \delta \left( \max \left[ W^{I,t}(w^{I,t}(0, N_i)), U \right] - W^{O,t}(w^{O,t}(y_N)) \right). \tag{A.2}
\]

Once a worker is employed, he becomes an insider, indexed by \( I \), and has employment protection. This protection (manifested through firing costs) implies that insiders will receive a higher wage when wages are renegotiated. The value of being employed as an insider at a firm with productivity \( y \) is given by,

\[
r^{W,I,t}(w^{I,t}(y_N)) = w^{I,t}(y_N) + \delta \left( \max \left[ W^{I,t}(w^{I,t}(0, N_i)), U \right] - W^{I,t}(w^{I,t}(y_N)) \right). \tag{A.3}
\]

The value of being employed depends on whether the surplus of the match is negative if the firm is hit by a productivity shock. If it is negative, wage negotiations will fail and the worker will be laid off. However, if the surplus of a match is positive even if a productivity shock hits, then wages will be renegotiated and the value of being employed changes to,

\[
\begin{align*}
& \quad r^{W,I,R}(w^{I,R}(0, N_i)) = \begin{cases} 
  w^{I,R}(0, N_i) + \eta \left( W^{I,R}(w^{I,R}(y_i, N_i)) - W^{I,R}(w^{I,R}(0, N_i)) \right) + \lambda d (U - W^{I,R}(w^{I,R}(0, N_i))), & \text{if } t = R, \\
  + \lambda d (U - W^{I,R}(w^{I,R}(0, N_i))), & \text{if } t = B.
\end{cases} \tag{A.4}
\end{align*}
\]

\[
\begin{align*}
& \quad r^{W,I,B}(w^{I,B}(0, N_i)) = \begin{cases} 
  w^{I,B}(0, N_i) + g(\varphi) \left( W^{I,B}(w^{I,B}(y_i, N_i)) - W^{I,B}(w^{I,B}(0, N_i)) \right) + \lambda d (U - W^{I,B}(w^{I,B}(0, N_i))), & \text{if } t = R, \\
  \lambda d (U - W^{I,B}(w^{I,B}(0, N_i))), & \text{if } t = B.
\end{cases} \tag{A.5}
\end{align*}
\]

The value of being employed at a firm, which has been hit by a productivity shock \( (y_i = 0) \), depends on the wage, the type \( t \in \{B, R\} \) of a firm, the respective rate at which the firm is able
to restore the productivity $y_t$ from 0 to $y$, that is, $\eta$ for firms that do their own research and $g(\varphi)$ for firms that buy the innovation, and on the destruction rate $\lambda_d$.

**Firms:** Firms that specialize in innovation will not be active on the labor market for production workers, i.e., $N_t^S = 0$. Thus, labor market conditions only enter the expected profit of a selling firm via the prices it receives for its innovation. The expected profit of a type $S$ firm that is doing research to obtain a new innovation is given by,

\[
(r + \lambda_s) \pi^S (0, 0, k_i) = -k_i + \eta \left( \pi^S (0, y, k_i) - \pi^S (0, 0, k_i) \right). \tag{A.6}
\]

The expected profit of a type $S$ firm, which sells its innovation, is given by,

\[
r \pi^S (0, y, k_i) = \varphi g(\varphi) \max \left[ E_{N_j} \left[ p \left( k_i, N_j^B \right) \right] + \pi^S (0, 0, k_i) - \pi^S (0, y, k_i), 0 \right] \tag{A.7}
\]

\[
+ \delta \left( \pi^S (0, 0, k_i) - \pi^S (0, y, k_i) \right),
\]

where the price $p \left( k_i, N_j^B \right)$ that is negotiated on the innovation market will depend on the surplus that is generated. The surplus will depend on the innovation cost $k_i$ of the seller and the number of workers employed at the buyer $N_j^B$. The buyer’s innovation cost does not enter the surplus, since a firm that decided to buy the input $y$ will also do so in the future, that is, it will never decide to do own research. Sellers only sell their innovation when the surplus is positive.

Let us now consider firms, which have been hit by a productivity shock, i.e., $y_i = 0$, and laid off $L_i^t$ workers. The respective Bellman equations are therefore given by,

\[
(r + \lambda_d) \pi^R (N_i^R, L_i^R, 0, k_i) = -w_i^{l,R} (0, N_i^R - L_i^R) \left( N_i^R - L_i^R \right) - k_i
\]

\[
+ \eta \left( \max_H \left[ \pi^R \left( N_i^R - L_i^R + H, L_i^R, y, k_i \right) - \frac{c}{\lambda_m (\theta)} H \right] - \pi^R \left( N_i^R, L_i^R, 0, k_i \right) \right),
\]

\[
(r + \lambda_d) \pi^B (N_i^B, L_i^B, 0, k_i) = -w_i^{l,B} (0, N_i^B - L_i^B) \left( N_i^B - L_i^B \right) + g(\varphi) \int_0^{k_{\text{max}}} \max \left[ S_i^{B_0}, 0 \right] h(k_j) dk_j,
\]

where the surplus from buying an innovation or product innovation $S_i^B$ in the innovation market is given by,

\[
S_i^B = \max_H \left[ \pi^B \left( N_i^B - L_i^B + H, L_i^B, y, k_i \right) - \frac{c}{\lambda_m (\theta)} H \right] - \pi^B \left( N_i^B, L_i^B, 0, k_i \right) - p \left( k_j, N_i^B - L_i^B \right).
\]

Since the marginal value of a newly hired worker is the same irrespective of whether the firm just entered the labor market or restored its productivity after a productivity shock $\delta$, i.e.,

\[
\frac{\partial \pi^t (N_i^t, L_i^t, y, k_i)}{\partial N_i^t} = \frac{\partial \pi^t (N_i^t - L_i^t + H, L_i^t, y, k_i)}{\partial H},
\]

26
firms, which have laid off \( L_i^t \) workers, will, once they have restored their productivity, hire \( L_i^t \) workers to restore their optimal employment level \( N_i^t \) by posting \( V_i^t = L_i^t/\lambda_m(\theta) \) vacancies. Note further, that the wages \( w^{R,R}(0,\cdot) \) and \( w^{B,B}(0,\cdot) \) that the \( N_i^t - L_i^t \) insiders receive as along as the firm employs them without having a product idea, i.e., \( y_i = 0 \), are independent of the number of workers employed. The reason is that the marginal product is zero and recruiting costs linear in \( N_i^t \). The exact wage expressions are derived in Appendix Appendix B. There will therefore be \( N_i^t - L_i^t \) insiders and \( L_i^t \) outsiders once the firm restarts production. This gives the following Bellman equation,

\[
n\pi^t(N_i^t, L_i^t, y, k_i) = y(N_i^t)^\alpha - w^{I,t}(y, N_i^t) N_i^t + \gamma (r + \delta) f L_i^t + \delta \max_L \left[ \pi^t(N_i^t, L, 0, k_i) - f L \right] - \pi^t(N_i^t, L_i^t, y, k_i).\]

From Appendix Appendix B we know that wage difference between insiders and outsider is given by \( \gamma (r + \delta) f \). So we can rewrite the wages of outsiders as \( w^{a,t}(y, N_i^t) = w^{I,t}(y, N_i^t) - \gamma (r + \delta) f \).

The firm can choose any number \( L \) of workers it wants to lay off. The respective first order condition for type \( R \) and \( B \) firms are given by

\[
\frac{\partial \pi^R(N_i^t, L_i^t, 0, k_i)}{\partial L_i^t} - f = \frac{w^{R,R}(0,\cdot) + \eta \frac{\partial \pi^R(N_i^t, L_i^t, y, k_i)}{\partial L_i^t}}{r + \lambda_d + \eta} - \frac{c}{\lambda_m(\theta)} - f,
\]

\[
\frac{\partial \pi^{O,l,b}(N_i^t, L_i^t, 0, k_i)}{\partial L_i^t} - f = \frac{w^{B,B}(0,\cdot) + g(\varphi)(1 - \beta) \left( \frac{\partial \pi^{O,l,b}(N_i^t, L_i^t, y, k_i)}{\partial L_i^t} - \frac{c}{\lambda_m(\theta)} \right)}{r + \lambda_d + g(\varphi)(1 - \beta)} - f.
\]

Since wages \( w^{R,R}(0,\cdot) \) and \( w^{B,B}(0,\cdot) \) are independent of the number of employed (and laid off workers), it follows that the firm either lays off all workers or none.

Given the result that \( L_i^t = 0 \) or \( L_i^t = N_i^t \) we can write the following Bellman equations for these two corner solutions. The following Bellman equation,

\[
r\pi^{h,t}(N_i^t, y, k_i) = y(N_i^t)^\alpha - w^{h,t}(y, N_i^t) N_i^t + \delta \max \left[ \pi^{h,t}(N_i^t, 0, k_i), \pi^{O,t}(0, 0, k_i) - f N_i^t \right] - \pi^{h,t}(N_i^t, y, k_i), \tag{A.8}
\]

\[
27
\]
characterizes the flow profit of a type $B$ or $R$ firm with productivity $y$, innovation cost $k_i$, and $N_i^t$ workers. The Bellman equations for a type $R$ firm that decides to do own research when it was hit by a productivity shock are given by,

$$(r + \lambda d) \pi^{O,R}(0,0,k_i) = -k_i + \eta \left( \pi^{O,R}(N_i^R, y, k_i) - \frac{c}{\lambda_m(\theta)} N_i^R - \pi^{O,R}(0,0,k_i) \right),$$

(A.9)

$$(r + \lambda d) \pi^{I,R}(N_i^R,0,k_i) = -w^{I,R}(0,N_i^R) N_i^R - k_i + \eta \left( \pi^{I,R}(N_i^R, y, k_i) - \pi^{I,R}(N_i^R,0,k_i) \right),$$

(A.10)

with and without laying off workers, respectively. A type $B$ firm that decides to acquire an innovation when it is without one has the following expected profit,

$$(r + \lambda d) \pi^{O,B}(0,0,k_i) = g(\varphi) \int_{0}^{k_{max}} \max[S^B,0] h(k_j) dk_j,$$

(A.11)

$$(r + \lambda d) \pi^{I,B}(N_i^B,0,k_i) = -w^{I,B}(0,N_i^B) N_i^B + g(\varphi) \int_{0}^{k_{max}} \max[S^B,0] h(k_j) dk_j,$$

(A.12)

with and without laying off workers, respectively, where the surplus for the buyer is given by,

$$S^B = \begin{cases} 
\pi^{O,B}(N_i^B, y, k_i) - \frac{c}{\lambda_m(\theta)} N_i^B - \pi^{O,B}(0,0,k_i) - p(k_j,0) & \text{if } L_i^B = N_i^B, \\
\pi^{I,B}(N_i^B, y, k_i) - \pi^{I,B}(N_i^B,0,k_i) - p(k_j,N_i^B) & \text{if } L_i^B = 0.
\end{cases}$$

The decision whether to do own research or, instead, acquire an innovation depends on the rate $\eta$ or $g(\varphi)$ at which the firm can restore its productivity level, on the level of firm-specific innovation cost $k_i$, and the expected price of the innovation. Since a firm can buy an innovation only from firms that decide to sell their innovations, we denote by $h(k_j)$ the pdf of those firms that are willing to sell their innovations and that have innovation cost $k_j$ (in equilibrium $h(k_j) = \xi(k_j)/\Xi(k^*)$, since all firms with $k_j$ below some threshold $k^*$ prefer to specialize in innovation and are willing to sell their innovations). The maximum operator in the integral guarantees that firms will buy an innovation only when the surplus is positive.

The marginal value of an additional worker for a firm with productivity $y$ is given by differentiating equation (A.8). The value depends on whether a firm lays off workers when it is hit
by a productivity shock, i.e.,

\[
\frac{\partial \pi^{h,t}(N_t^i, y, k_i)}{\partial N_t^i} = \begin{cases} 
\alpha y (N_t^i)^{\alpha - 1} - w^{h,t}(y, N_t^i) - \frac{\partial w^{h,t}(y, N_t^i)}{\partial N_t^i} N_t^i + \delta \frac{\partial \pi^{I,t}(0, 0, k_i)}{\partial N_t^i} & \text{if } L_t^i = 0, \\
\alpha y (N_t^i)^{\alpha - 1} - w^{h,t}(y, N_t^i) - \frac{\partial w^{h,t}(y, N_t^i)}{\partial N_t^i} N_t^i + \delta \frac{\partial \pi^{O,t}(0, 0, k_i)}{\partial N_t^i} - \delta f & \text{if } L_t^i = N_t^i,
\end{cases}
\]

(A.13)

The third term in the marginal value of an additional worker \(\partial w^{O,t}(y, N_t^i) / \partial N_t^i\) captures the fact that each time a new worker is hired, the wages of all workers are renegotiated and adjusted to the new marginal revenue product.

The marginal value of an additional worker for a firm, which has been hit by a productivity shock but retains its workers, can be obtained by differentiating equations (A.10) and (A.12) and using equation (C.2) to substitute out the price for an innovation (see Appendix Appendix C below), i.e.,

\[
\frac{\partial \pi^{I,R}(N_t^R, 0, k_i)}{\partial N_t^R} = \frac{-w^{I,R}(0, N_t^R) - \frac{\partial w^{I,R}(0, N_t^R)}{\partial N_t^R} N_t^R + \eta \frac{\partial \pi^{I,R}(N_t^R, y, k_i)}{\partial N_t^R}}{(r + \lambda d + g(\varphi)(1 - \beta))},
\]

(A.14)

\[
\frac{\partial \pi^{I,B}(N_t^B, 0, k_i)}{\partial N_t^B} = \frac{-w^{I,B}(0, N_t^B) - \frac{\partial w^{I,B}(0, N_t^B)}{\partial N_t^B} N_t^B + g(\varphi)(1 - \beta) \frac{\partial \pi^{I,B}(N_t^B, y, k_i)}{\partial N_t^B}}{(r + \lambda d + g(\varphi)(1 - \beta))}.
\]

(A.15)

The marginal value of an additional worker for a firm, which has been hit by a productivity shock but lays off its workers, can be obtained by differentiating equations (A.9) and (A.11) and using equation (C.2) to substitute out the price for an innovation (see Appendix Appendix C below), i.e.,

\[
\frac{\partial \pi^{O,R}(0, 0, k_i)}{\partial N_t^R} = \frac{1}{r + \lambda d + \eta} \left( \frac{\partial \pi^{O,R}(N_t^R, y, k_i)}{\partial N_t^R} - \frac{c}{\lambda_m(\theta)} \right) = 0,
\]

(A.16)

\[
\frac{\partial \pi^{O,B}(0, 0, k_i)}{\partial N_t^B} = \frac{g(\varphi)(1 - \beta)}{(r + \lambda d + g(\varphi)(1 - \beta))} \left( \frac{\partial \pi^{O,B}(N_t^B, y, k_i)}{\partial N_t^B} - \frac{c}{\lambda_m(\theta)} \right) = 0.
\]

(A.17)

The vacancy creation condition (2) implies that these values are equal to zero.
Appendix B. Wage equations

Let us first consider the wages paid in type $R$ firms, if no workers are laid off in case a productivity shock hits ($L_i^R = 0$). Wage bargaining according to equations (6) and (7) implies the following surplus splitting rule for outsiders in firms with $y_i = y$, for insiders in firms with $y_i = y$ and for insiders in firms with $y_i = 0$ (since firms with $y_i = 0$ never hire workers, we do not need to consider outsiders in firms with $y_i = 0$),

$$(1 - \gamma) \left( W^{O,R} \left( w^{O,R} (y, N_i^R) \right) - U \right) = \gamma \left( \frac{\partial \pi^{O,R} (N_i^R, y, k_i)}{\partial N_i^R} \right),$$

$$(1 - \gamma) \left( W^{I,R} \left( w^{I,R} (y, N_i^R) \right) - U \right) = \gamma \left( \frac{\partial \pi^{I,R} (N_i^R, y, k_i) + f}{\partial N_i^R} \right),$$

$$(1 - \gamma) \left( W^{I,R} \left( w^{I,R} (0, N_i^R) \right) - U \right) = \gamma \left( \frac{\partial \pi^{I,R} (N_i^R, 0, k_i) + f}{\partial N_i^R} \right),$$

where firms only have to pay firing costs $f$, if they do not continue to employ an insider.

Substituting the marginal value of employing a worker in the respective situation from equations (A.13) and (A.14) and the workers’ surplus from employment using equations (A.2) and (A.4), i.e.,

$$[W^{O,R} \left( w^{O,R} (y, N_i^R) \right) - U] = \frac{w^{O,R} (y, N_i^R) - rU + \delta \left[ W^{I,R} \left( w^R (0, N_i) \right) - U \right]}{(r + \delta)},$$

$$[W^{I,R} \left( w^{I,R} (y, N_i^R) \right) - U] = \frac{w^{I,R} (y, N_i^R) - rU + \delta \left[ W^{I,R} \left( w^R (0, N_i) \right) - U \right]}{(r + \delta)},$$

$$[W^{I,R} \left( w^{I,R} (0, N_i^R) \right) - U] = \frac{w^{I,R} (0, N_i) - rU + \eta \left[ W^{I,R} \left( w^R (0, N_i) \right) - U \right]}{(r + \eta + \lambda d)},$$

and rearranging using again the surplus splitting rules in the equations above, leads to the following differential wage equations,

$$w^{O,R} (y, N_i^R) = (1 - \gamma) rU + \gamma \left( \alpha y (N_i^R)^{\alpha - 1} - \frac{\partial w^{O,R} (y, N_i^R)}{\partial N_i^R} N_i^R \right) - \gamma \delta f,$$

$$w^{I,R} (y, N_i^R) = (1 - \gamma) rU + \gamma \left( \alpha y (N_i^R)^{\alpha - 1} - \frac{\partial w^{I,R} (y, N_i^R)}{\partial N_i^R} N_i^R \right) + \gamma \tau f,$$

$$w^{I,R} (0, N_i^R) = (1 - \gamma) rU - \frac{\partial w^{I,R} (0, N_i^R)}{\partial N_i^R} N_i^R + \gamma (r + \lambda_d) f,$$

Solving the differential equations for $w^{O,R} (y, N_i^R)$ and $w^{I,R} (y, N_i^R)$ following Cahuc and Wasmer (2001) and Cahuc, Marque, and Wasmer (2008) gives the wages paid to outsiders and
insiders at type $R$ firms with $y_i = y$,

$$w^{O,R}(y, N_i^{R}) = (1 - \gamma) z + \gamma \theta c + \gamma \frac{\alpha}{1 - \gamma (1 - \alpha)} y (N_i^{R})^{\alpha - 1} - \gamma \delta f, \quad (B.1)$$

$$w^{I,R}(y, N_i^{R}) = (1 - \gamma) z + \gamma \theta c + \gamma \frac{\alpha}{1 - \gamma (1 - \alpha)} y (N_i^{R})^{\alpha - 1} + \gamma rf, \quad (B.2)$$

where we substituted the value of being unemployed by $(1 - \gamma) r U = (1 - \gamma) z + \gamma \theta c$. Type $R$ firms without a product idea, i.e., $y_i = 0$, that keep their workers, i.e., $L_i = 0$, pay the following wage,

$$w^{I,R}(0, N_i^{R}) = (1 - \gamma) z + \gamma \theta c + \gamma (r + \lambda_d) f, \quad (B.3)$$

since the differential wage equation for $w^{I,R}(0, N_i^{R})$ is independent of $N_i^{R}$ and is therefore given by setting $\partial w^{I,R}(0, N_i^{R}) / \partial N_i^{R} = 0$.

Now consider wages paid by type $B$ firms, if no workers are laid off in case a productivity shock hits ($L_i^B = 0$). The surplus splitting rules are given by,

$$(1 - \gamma) (W^{O,B}(w^{O,B}(y, N_i^{B})) - U) = \frac{\partial \pi^{O,B}(N_i^{B}, y, k_i)}{\partial N_i^{B}}),$$

$$(1 - \gamma) (W^{I,B}(w^{I,B}(y, N_i^{B})) - U) = \frac{\partial \pi^{I,B}(N_i^{B}, y, k_i)}{\partial N_i^{B}} + f),$$

$$(1 - \gamma) (W^{I,B}(w^{I,B}(0, N_i^{B})) - U) = \frac{\partial \pi^{I,B}(N_i^{B}, 0, k_i)}{\partial N_i^{B}} + f),$$

where the surplus splitting rule for the case with $y_i = 0$ takes into account that innovation price bargaining implies that part of the marginal value of continuing the employment relationship (the fraction $\beta$) is going to the seller. This causes the additional term in the last equation.

Substituting the marginal values of employing a worker in the respective situation from equations (A.13) and (A.15) and the workers’ surplus from employment,

$$[W^{O,B}(w^{O,B}(y, N_i^{B})) - U] = \frac{w^{O,B}(y, N_i^{B}) - r U + \delta [W^{I,B}(w^{I,B}(0, N_i^{B})) - U]}{(r + \delta)}$$

$$[W^{I,B}(w^{I,B}(y, N_i^{B})) - U] = \frac{w^{I,B}(y, N_i^{B}) - r U + \delta [W^{I,B}(w^{I,B}(0, N_i^{B})) - U]}{(r + \delta)}$$

$$[W^{I,B}(w^{I,B}(0, N_i^{B})) - U] = \frac{w^{I,B}(0, N_i^{B}) - r U + g(\varphi) [W^{I,B}(w^{I,B}(y, N_i^{B})) - U]}{(r + \lambda_d + g(\varphi))},$$

31
implies the following differential wage equations,

\[ w^{O,B}(y, N^B_i) = (1 - \gamma) rU + \gamma \alpha y (N^B_i)^{\alpha-1} - \gamma \frac{\partial w^{O,B}(y, N^B_i)}{\partial N^B_i} N^B_i - \delta \gamma f \]

\[ + \delta \frac{\beta g(\varphi)}{r + \lambda_d + g(\varphi)} \gamma \left( \frac{\partial \pi^{I,B}(N^B_i, 0, k_i)}{\partial N^B_i} + f \right), \]

\[ w^{I,B}(y, N^B_i) = (1 - \gamma) rU + \gamma \alpha y (N^B_i)^{\alpha-1} - \gamma \frac{\partial w^{I,B}(y, N^B_i)}{\partial N^B_i} N^B_i + \gamma r f \]

\[ + \delta \frac{g(\varphi) \beta}{r + \lambda_d + g(\varphi)} \gamma \left( \frac{\partial \pi^{I,B}(N^B_i, 0, k_i)}{\partial N^B_i} + f \right), \]

\[ w^{I,B}(0, N^B_i) = (1 - \gamma) rU - \gamma \frac{\partial w^{I,B}(0, N^B_i)}{\partial N^B_i} N^B_i + \gamma (r + \lambda_d) f \]

\[ - \beta g(\varphi) \gamma \left( \frac{\partial \pi^{I,B}(y, N^B_i, k_i)}{\partial N^B_i} + f \right), \]

where the last term in each wage equation appears due to innovation price bargaining. Since wages of outsiders and insiders at a firm with \( y_i = y \) only differ in a constant, we know that \( \partial w^{O,B}(y, N^B_i) / \partial N^B_i = \partial w^{I,B}(y, N^B_i) / \partial N^B_i \). This allows us to write the differences in wages between outsiders and insiders as,

\[ w^{I,B}(y, N^B_i) - w^{O,B}(y, N^B_i) = \gamma (r + \delta) f. \]

Substituting allows us to write the difference in the marginal values of employing an outsider and an insider as,

\[ \frac{\partial \pi^{O,B}(N^B_i, y, k_i)}{\partial N^B_i} - \frac{\partial \pi^{I,B}(N^B_i, y, k_i)}{\partial N^B_i} = \frac{w^{I,B}(y, N^B_i) - w^{O,B}(y, N^B_i)}{(r + \delta)} = \gamma f. \]

Given the vacancy creation condition, we can write the marginal value of employing an insider as

\[ \frac{\partial \pi^{I,B}(N^B_i, y, k_i)}{\partial N^B_i} = \frac{\partial \pi^{O,B}(N^B_i, y, k_i)}{\partial N^B_i} - \gamma f = \frac{c}{\lambda_m(\theta)} - \gamma f. \]

This allows us to determine the wage for an insider at a firm with \( y_i = 0 \),

\[ w^{I,B}(0, N^B_i) = (1 - \gamma) rU + \gamma (r + \lambda_d) f - \beta g(\varphi)(\frac{c}{\lambda_m(\theta)} + (1 - \gamma) f) \]

where we used the fact that the differential equation is independent of \( N^B_i \), i.e., \( \partial w^{I,B}(0, N^B_i) / \partial N^B_i = 0 \).

Substituting implies that the marginal value of employing an insider with \( y_i = 0 \) is independent of the number of employed workers, i.e.,

\[ \frac{\partial \pi^{I,B}(N^B_i, 0, k_i)}{\partial N^B_i} = \frac{g(\varphi)(1 - \beta)(\frac{c}{\lambda_m(\theta)} - \gamma f) - w^{I,B}(0, N^B_i)}{(r + \lambda_d + g(\varphi)(1 - \beta))}. \]
This allows us to write the wage equation for an outsider and an insider at a firm with $y_i = y$ as,

\[
w^{O,B} (y, N_i^B) = (1 - \gamma) r U + \gamma \frac{\alpha}{1 - \gamma (1 - \alpha)} y (N_i^B)^{\alpha - 1} - \delta \gamma f \\
+ \delta \beta g (\varphi) (1 - \beta) \left( \frac{c}{\lambda_m (\theta)} - \gamma f \right) - w^{I,B} (0, N_i^B)
\]

\[
w^{I,B} (y, N_i^B) = (1 - \gamma) r U + \gamma \frac{\alpha}{1 - \gamma (1 - \alpha)} y (N_i^B)^{\alpha - 1} + \gamma r f \\
+ \delta \beta g (\varphi) (1 - \beta) \left( \frac{c}{\lambda_m (\theta)} - \gamma f \right) - w^{I,B} (0, N_i^B)
\]

If we substituted the value of being unemployed by $(1 - \gamma) r U = (1 - \gamma) z + \gamma \theta c$, then we get wages as a function of labor market tightness.

Now consider wages paid by type $B$ and $R$ firms, if all workers are laid off in case a productivity shock hits ($L_i^t = N_i^t$). If firms lay off their workforce when they are hit by a productivity shock, i.e., $L_i^t = N_i^t$, then workers do not have a chance to renegotiate their wages as insiders. This implies that there will be only one wage, if $L_i^t = N_i^t$. The fact that workers are paid only the outsider wage can also be seen by noting that the workers’ value of being employed in Equation (A.2) simplifies, since $\max [W^{O,t} (w^{O,t} (y, N_i)) , U] = U$. The workers’ surplus from employment is hence given by,

\[
W^{O,t} (w^{O,t} (y, N_i)) - U = \frac{w^{O,t} (y, N_i) - r U}{(r + \delta)}.
\]

The surplus splitting rule is given by

\[
(1 - \gamma) \left( W^{O,t} (w^{O,t} (y, N_i)) - U \right) = \gamma \left( \frac{\partial \pi^{O,t} (N_i^t, y, k_i)}{\partial N_i^t} \right).
\]

Substituting the marginal value of employing a worker from equations (A.13) implies that the respective wage is given by,

\[
w^{O,t} (y, N_i^t) = (1 - \gamma) z + \gamma \theta c + \frac{\alpha}{1 - \gamma + \gamma \alpha} y (N_i^t)^{\alpha - 1} - \gamma \delta f, \text{ all for } t \in \{B, R\}. \quad (B.4)
\]

Note, that the wage in this case is independent of the firm’s decision to either do own research or buy a product idea, since the vacancy creation conditions and therefore the level of employment are identical.
Appendix C. Innovation price

The vacancy creation and firing conditions (2), (4) and (5) imply that in a given steady state, all type B firms have either 0 or $N_j^B$ employees. This simplifies the analysis and implies that the expected price of an innovation charged by a firm with innovation cost $k_i$ is given by $p(k_i, 0)$ or $p(k_i, N_j^B)$. Since we concentrate on the parameter sets that guarantee the existence of an innovation market, we know that all type S firms are willing to sell to any type B firm, that is, all matches in the innovation market will generate a positive surplus.

The innovation price is given by the surplus splitting rule,

$$p(k_j, 0) = \beta \left( \pi^{O,B}(N_i^B, y, k_i) - \frac{e}{\lambda_m(\theta)} N_i^B - \pi^{O,B}(0, 0, k_i) \right) + (1 - \beta) \left( \pi^S(0, y, k_j) - \pi^S(0, 0, k_j) \right),$$

$$p(k_j, N_i^B) = \beta \left( \pi^{f,B}(N_i^B, y, k_i) - \pi^{f,B}(N_i^B, 0, k_i) \right) + (1 - \beta) \left( \pi^S(0, y, k_j) - \pi^S(0, 0, k_j) \right).$$

The closed form expressions for the expected profit of type S firms that sell their innovations and of type B firms that buy innovations are as follows. Given the fact that the price of a type S firm with innovation cost $k_i$ is given by $p(k_i, 0)$ or $p(k_i, N_j^B)$, respectively, and using equations (A.7) and (A.6) the expected profit with $y_i \in \{0, y\}$ can be written as,

$$\pi^S(0, y, k_i) = \frac{(r + \lambda_s + \eta) \varphi g(\varphi) p(k_i, N) - (\delta + \varphi g(\varphi)) k_i}{r (r + \lambda_s + \eta) + (\delta + \varphi g(\varphi)) (r + \lambda_s)},$$

$$\pi^S(0, 0, k_i) = \frac{\eta \varphi g(\varphi) p(k_i, N) - (r + \varphi g(\varphi)) k_i}{r (r + \lambda_s + \eta) + (\delta + \varphi g(\varphi)) (r + \lambda_s)},$$

where $N = N_j^B$ if $L_j^B = 0$ and $N = 0$ if $L_j^B = N_j^B$. Using equations (A.8) and (A.12) the expected profit for firms that do not lay off their workers if they are hit by a productivity shock can be written as,

$$\pi^{f,B}(N_i^B, y, k_i) = \frac{(r + \lambda_d + g(\varphi)) (y (N_i^B)^{\alpha} - w^{I,B}(y, N_i^B) N_i^B)}{(r + \lambda_d)(r + \delta) + g(\varphi) r}$$

$$- \delta \frac{w^{I,B}(0, N_i^B) N_i^B + g(\varphi) E_{k_j} [p(k_j, N_i^B)]}{(r + \lambda_d)(r + \delta) + g(\varphi) r},$$

$$\pi^{f,B}(N_i^B, 0, k_i) = \frac{g(\varphi) (y (N_i^B)^{\alpha} - w^{I,B}(y, N_i^B) N_i^B)}{(r + \lambda_d)(r + \delta) + g(\varphi) r}$$

$$- (r + \delta) \frac{w^{I,B}(0, N_i^B) N_i^B + g(\varphi) E_{k_j} [p(k_j, N_i^B)]}{(r + \lambda_d)(r + \delta) + g(\varphi) r}.$$
If workers are laid off in case of a productivity shock, the expected profits are given by,

\[
\pi^{O,B}(N^B_i, y, k_i) = \frac{(r + \lambda_d + g(\varphi))(y(N^B_i)^\alpha - w^{O,B}(y, N^B_i) N^B_i - \delta f N^B_i)}{(r + \lambda_d)(r + \delta) + g(\varphi) r} - \frac{\delta g(\varphi)}{(r + \lambda_d)(r + \delta) + g(\varphi) r} \left( \frac{c}{\lambda_m(\theta)} N^B_i + E_{k_i}[p(k_i, 0)] \right),
\]

\[
\pi^{O,B}(0, 0, k_i) = \frac{g(\varphi)(y(N^B_i)^\alpha - w^{O,B}(y, N^B_i) N^B_i - \delta f N^B_i)}{(r + \lambda_d)(r + \delta) + g(\varphi) r} - \frac{(r + \delta) g(\varphi)}{(r + \lambda_d)(r + \delta) + g(\varphi) r} \left( \frac{c}{\lambda_m(\theta)} N^B_i + E_{k_i}[p(k_i, 0)] \right).
\]

To determine the price that type $B$ firms expect to pay for an innovation, we first focus on the average seller that has innovation cost $\bar{k}$ such that its price equals the expected price, i.e.,

\[p(\bar{k}, N^B_i) = E_{k_i}[p(k, N^B_i)] \text{ or } p(\bar{k}, 0) = E_{k_i}[p(k, 0)].\]

Computing the differences in expected profits using equations (C.3) to (C.8) and plugging the results into the innovation price equations (C.1) and (C.2) leads to

\[
E_{k_i}[p(\bar{k}, N^B_i)] = \frac{K_2 \beta (r + \lambda_d) \left( y(N^B_i)^\alpha - w^{B}(y, N^B_i) N^B_i \right) + K_2 \beta r w t^{B}(0, N^B_i) N^B_i}{K_1 K_2 - K_1 (1 - \beta) (r + \lambda_s) \varphi g(\varphi) - K_2 \beta r g(\varphi)} \]

\[+ \frac{K_1 (1 - \beta) \varphi \bar{k}}{K_1 K_2 - K_1 (1 - \beta) (r + \lambda_s) \varphi g(\varphi) - K_2 \beta r g(\varphi)},\]

\[
E_{k_i}[p(\bar{k}, 0)] = \frac{K_2 \beta (r + \lambda_d) \left( y(N^B_i)^\alpha - w^{O,B}(y, N^B_i) N^B_i - \delta f N^B_i - (r + \delta) \frac{c}{\lambda_m(\theta)} N^B_i \right)}{K_1 K_2 - K_1 (1 - \beta) (r + \lambda_s) \varphi g(\varphi) - K_2 \beta r g(\varphi)} \]

\[+ \frac{K_1 (1 - \beta) \varphi \bar{k}}{K_1 K_2 - K_1 (1 - \beta) (r + \lambda_s) \varphi g(\varphi) - K_2 \beta r g(\varphi)},\]

where

\[K_1 = (r + \delta) (r + \lambda_d) + r g(\varphi),\]

\[K_2 = (r + \delta + \varphi g(\varphi)) (r + \lambda_s) + r \eta.\]

Given the expected price in equation (C.9) or (C.10) the innovation price $p(k, N^B_i)$ or $p(k, 0)$ for a seller with innovation cost $k$ is given by substituting the expected price in the respective expected profit functions (C.3) to (C.8) and inserting them into the innovation price equations
(C.1) and (C.2). Rearranging implies,

\[
p (k_j, N^B_i) = \frac{K_2 \beta (r + \lambda_d) \left( y \left( N^B_i \right)^\alpha - w^{I,B} \left( y, N^B_i \right) N^B_i \right) + K_2 \beta r w^B (0, N^B_i) N^B_i}{K_1 K_2 - K_1 (1 - \beta) (r + \lambda_s) \varphi g (\varphi)} \\
+ \frac{K_1 (1 - \beta) r k_j + K_2 \beta r g (\varphi) p (k, N^B_i)}{K_1 K_2 - K_1 (1 - \beta) (r + \lambda_s) \varphi g (\varphi)},
\]

\[
p (k_j, 0) = \frac{K_2 \beta (r + \lambda_d) \left( y \left( N^B_i \right)^\alpha - w^{O,B} \left( y, N^B_i \right) N^B_i - \delta f N^B_i - (r + \delta) \frac{c}{\lambda_m (\theta)} N^B_i \right)}{K_1 K_2 - K_1 (1 - \beta) (r + \lambda_s) \varphi g (\varphi)} + \frac{K_1 (1 - \beta) r k_j + K_2 \beta r g (\varphi) p (k, 0)}{K_1 K_2 - K_1 (1 - \beta) (r + \lambda_s) \varphi g (\varphi)}.
\]

Appendix D. Vacancy creation conditions

First, we consider the vacancy creation conditions for type \( R \) and \( B \) firms that do not lay off workers when they are hit by a productivity shock, i.e., if \( L^I_i = 0 \). Using the respective marginal values of a worker from section Appendix B and the fact that,

\[
\frac{\partial \pi^{O,I} (N^I_i, y, k_i)}{\partial N^I_i} - \frac{\partial \pi^{I,t} (N^I_i, y, k_i)}{\partial N^I_i} = \frac{w^{I,t} (y, N^I_i) - w^{O,t} (y, N^I_i)}{(r + \delta)} = \gamma f,
\]

we can write for type \( R \) firms,

\[
\frac{\partial \pi^{I,R} (N^R_i, y, k_i)}{\partial N^R_i} = \frac{(1 - \gamma) \alpha}{1 - \gamma (1 - \alpha)} \frac{y \left( N^R_i \right)^{\alpha-1} - (1 - \gamma) r U - \gamma r f + \delta \frac{\partial \pi^{I,R} (N^R_i, 0, k_i)}{\partial N^R_i}}{(r + \delta)}
\]

\[
= \frac{(1 - \gamma) \alpha}{1 - \gamma (1 - \alpha)} \frac{y \left( N^R_i \right)^{\alpha-1} - (1 - \gamma) r U - \gamma r f + \delta \eta \frac{c}{\lambda_m (\theta)} + (1 - \gamma) f}{(r + \delta)} - w^{I,R} (0, N^R_i) - \eta f
\]

\[
+ \frac{\delta}{(r + \delta)} \frac{\eta \frac{c}{\lambda_m (\theta)} - (1 - \gamma) r U - \gamma (r + \lambda_d + \eta) f}{(r + \lambda_d + \eta)}.
\]

Using the fact that,

\[
\frac{\partial \pi^{O,t} (N^I_i, y, k_i)}{\partial N^I_i} = \frac{\partial \pi^{I,t} (N^I_i, y, k_i)}{\partial N^I_i} + \gamma f = \frac{c}{\lambda_m (\theta)},
\]

36
implies the following vacancy creation condition for type \( R \) firms, which do not lay off workers if a productivity shock hits,

\[
\frac{c}{\lambda_m(\theta)} = \frac{(1 - \gamma) \alpha y (N_i^R)^{\alpha - 1} - (1 - \gamma) rU}{(r + \delta)} + \frac{\delta}{(r + \delta)} \frac{\eta}{\lambda_m(\theta)} - (1 - \gamma) rU + \frac{\delta}{(r + \delta)} \frac{c}{\lambda_m(\theta)} (r + \lambda_d + \eta)
\]

Similarly for type \( B \) firms, i.e.,

\[
\frac{\partial \pi^{I,B}(N_i^B, y, k_i)}{\partial N_i^B} = \frac{(1 - \gamma) \alpha y (N_i^B)^{\alpha - 1} - (1 - \gamma) rU - (r + \delta + \lambda_d + \eta) (1 - \gamma) rU}{(r + \delta) (r + \lambda_d + r\eta)}
\]

Using the fact that,

\[
\frac{\partial \pi^{O,I}(N_i^I, y, k_i)}{\partial N_i^I} = \frac{\partial \pi^{I,I}(N_i^I, y, k_i)}{\partial N_i^I} + \gamma f = \frac{c}{\lambda_m(\theta)}.
\]

implies,

\[
\frac{c}{\lambda_m(\theta)} = \frac{(1 - \gamma) \alpha y (N_i^B)^{\alpha - 1} - (1 - \gamma) rU - \delta (1 - \gamma) f}{(r + \delta)} + \frac{\delta}{(r + \delta)} \frac{r + \lambda_d + g(\varphi) - g(\varphi) \gamma \beta}{r + \lambda_d + g(\varphi)} \left( \frac{\partial \pi^{I,B}(N_i^B, 0, k_i)}{\partial N_i^B} + f \right).
\]
\[
\frac{c}{\lambda_m(\theta)} = \frac{\frac{(1-\gamma)\alpha}{1-\gamma(1-\alpha)}y \left(N_t^B\right)^{\alpha-1} - (1-\gamma) rU - \delta (1-\gamma) f}{(r+\delta)} \\
+ \frac{\delta}{(r+\delta)} \frac{r + \lambda_d + g(\varphi) - g(\varphi) \gamma \beta}{(r + \lambda_d + g(\varphi))} \frac{g(\varphi) (1 - (1-\gamma) \beta)}{\lambda_m(\theta)} - (1-\gamma) rU \\
+ \frac{\delta}{(r+\delta)} \frac{r + \lambda_d + g(\varphi) - g(\varphi) \gamma \beta}{(r + \lambda_d + g(\varphi))} \frac{r + \lambda_d + g(\varphi) (1 - (1-\gamma) \beta)}{r + \lambda_d + g(\varphi) (1 - (1-\gamma) \beta)} (1-\gamma) f, \\
\]

Rearranging implies the following vacancy creation condition for type B firms, which do not lay off workers if a productivity shock hits,

\[
\frac{c}{\lambda_m(\theta)} = \frac{C_2}{C_1} \left( \frac{(1-\gamma)\alpha}{1-\gamma(1-\alpha)}y \left(N_t^B\right)^{\alpha-1} - (1-\gamma) rU \right) - \frac{r + \lambda_d + g(\varphi) - g(\varphi) \gamma \beta}{C_1} \delta (1-\gamma) rU \\
+ \frac{g(\varphi) \beta (1 - \gamma) g(\varphi) \gamma \beta}{C_1} \delta (1-\gamma) f,
\]

with

\[
C_1 = C_2 (r+\delta) - (r + \lambda_d + g(\varphi) - \gamma \beta g(\varphi)) \delta (1 - (1-\gamma) \beta) g(\varphi), \\
C_2 = (r + \lambda_d + g(\varphi)) (r + \lambda_d + g(\varphi) (1 - \beta)).
\]

In case firms lay off their workforce once a productivity shock hits, the vacancy creation condition can be derived by substituting the marginal value of employing a worker as outsider from equation (A.13) into equation (2), i.e.,

\[
\frac{c}{\lambda_m(\theta)} = \frac{(1-\gamma)\alpha}{1-\gamma+\gamma\alpha^\alpha y \left(N_t^B\right)^{\alpha-1} - (1-\gamma) z - \gamma \theta c - (1-\gamma) \delta f}{r+\delta}.
\]

The first-order condition for the optimal number of posted vacancies in equation (2) shows that vacancy posting costs always exceeds the marginal value of an additional worker for a firm.
that has been hit by a productivity shock, i.e.,

$$\frac{\partial \pi_{I,t}^I(N_t^i,0,k_i)}{\partial V_t^i} = \frac{\partial \pi_{I,t}^I(N_t^i,0,k_i)}{\partial N_t^i} \lambda_m(\theta) - c < 0,$$

as one can easily verify by substituting the marginal value of an additional worker using the fact that,

$$\frac{\partial \pi_{O,t}^O(N_t^i,y,k_i)}{\partial N_t^i} = \frac{\partial \pi_{I,t}^I(N_t^i,y,k_i)}{\partial N_t^i} + \gamma f = \frac{c}{\lambda_m(\theta)}.$$

Thus, firms that have been hit by a productivity shock never post vacancies.

**Appendix E. Type choice**

We show

$$\frac{\partial \pi^S(0,y,k_i)}{\partial k_i} < \frac{\partial \pi^O,R(N_R^i,y,k_i)}{\partial k_i} < \frac{\partial \pi^O,B(N_B^i,y,k_i)}{\partial k_i} = 0.$$ 

Note first that $\pi^O,B(N_B^i,y,k_i)$ is independent of $k_i$ as shown in equation (C.7), which implies $\partial \pi^O,B(N_B^i,y,k_i)/\partial k_i = 0$.

The closed form expression for the expected profit of type $R$ firms is obtained by rearranging equations (A.9), (A.10) and (A.8), i.e.,

$$\pi^O,R(N_R^i,y,k_i) = \begin{cases} 
(r + \lambda_d + \eta) \left( y \left( N_R^i \right)^\alpha - w^O,R \left( y, N_R^i \right) N_R^i \right) - \delta k_i & \text{if } L_R^i = 0, \\
\frac{((r + \lambda_d + \delta + \eta))}{\delta \eta} \left( N_R^i \right)^\alpha - \frac{\delta \eta}{w^O,R \left( N_R^i \right)} N_R^i - \delta f N_R^i - \delta k_i & \text{if } L_R^i = N_R^i,
\end{cases}$$

The expected profit is strictly decreasing in $k_i$, which makes it less attractive for high innovation cost firms to do own research if they are hit by a productivity shock.

Type $S$ firms that only innovate in order to sell their innovations obtain the expected profit
\[ \pi^S(0, y, k_i) \text{, where substituting the price } p(k_i, N) \text{ implies,} \]

\[ \begin{align*}
\pi^S(0, y, k_i) &= \frac{(r + \lambda_s + \eta) \varphi g(\varphi) \beta (r + \lambda_d) \left( y \left( N_j^B \right)^\alpha \right)}{(r + \lambda_d)(r + \delta) + g(\varphi) r} \left( (r + \lambda_s)(r + \delta + \beta \varphi g(\varphi)) + r\eta \right) \\
&\quad + \frac{(r + \lambda_s + \eta) \varphi g(\varphi) \beta \left( rw^B \left( 0, N_j^B \right) N_j^B + rg(\varphi) p \left( \kappa_i, N_j^B \right) \right)}{(r + \lambda_d)(r + \delta) + g(\varphi) r} \left( (r + \lambda_s)(r + \delta + \beta \varphi g(\varphi)) + r\eta \right) \\
&\quad - \frac{(\delta + \beta \varphi g(\varphi)) \left( \lambda_i \right)(r + \lambda_s + \eta) \varphi g(\varphi) \beta (r + \lambda_d) \left( y \left( N_j^B \right)^\alpha \right) - w^B \left( y, N_j^B \right) N_j^B - \delta f N_j^B}{(r + \lambda_d)(r + \delta) + r\eta} k_i \text{ if } L_j^B = 0 , \\
\pi^S(0, y, k_i) &= \frac{(r + \lambda_s + \eta) \varphi g(\varphi) \beta (r + \lambda_d) \left( y \left( N_j^B \right)^\alpha \right) - w^B \left( y, N_j^B \right) N_j^B - \delta f N_j^B}{(r + \lambda_d)(r + \delta) + r\eta} \left( (r + \lambda_s)(r + \delta + \beta \varphi g(\varphi)) + r\eta \right) \\
&\quad + \frac{(r + \lambda_s + \eta) \varphi g(\varphi) \beta (r + \lambda_d) \left( (r + \delta) \left( \frac{c}{\lambda m(\theta)} N_j^B \right) + rg(\varphi) p \left( \kappa_i, 0 \right) \right)}{(r + \lambda_d)(r + \delta + \beta \varphi g(\varphi)) + r\eta} k_i \text{ if } L_j^B = N_j^B ,
\end{align*} \]

The expected profit \( \pi^S(0, y, k_i) \) is strictly decreasing in \( k_i \). Comparing how the expected profit of type \( S \) and \( R \) firms change with the innovation cost \( k_i \) reveals,

\[ \begin{align*}
\frac{\partial \pi^S(0, y, k_i)}{\partial k_i} &= \frac{\partial \pi^{O,R}(N_i^R, y, k_i)}{\partial k_i} \\
&= \frac{\delta}{(r + \delta)(r + \lambda_d + \eta)} \left( 1 + \frac{\eta \delta}{(r + \lambda_d)(r + \delta) + r\eta} \right) - \frac{(\delta + \beta \varphi g(\varphi))}{(r + \lambda_d)(r + \delta + \beta \varphi g(\varphi)) + r\eta} \\
&= \frac{(\lambda_s - \lambda_d)(r\delta + \delta) - (r + \lambda_d + \eta) r\beta \varphi g(\varphi)}{(r + \lambda_d)(r + \delta) + r\eta)((r + \lambda_s)(r + \delta + \beta \varphi g(\varphi)) + r\eta)} < 0,
\end{align*} \]

since \( \lambda_s < \lambda_d \) by assumption.

**Appendix E.1. Steady State Measures**

**Firm Flows and Innovation Market Tightness:** Denote the measure of firms that exit the economy each period by \( m^e \), where the assumptions regarding the destruction of firms imply,

\[ m^e = \lambda_i m^S(0, 0) + \lambda_d \left( m^R(0, N_i^R) + m^B \left( 0, N_i^B \right) \right) . \]

In a steady state the measure of firms that exit the economy is equal to the measure of new firms that enter, i.e., \( m^e = m^n \). The respective measure of firms evolve according to the difference
between in- and outflows, i.e.,

\[ \dot{m}^S (0, 0) = (\delta + \varphi g (\varphi)) m^S (y, 0) - (\lambda_s + \eta) m^S (0, 0) \]  
\( \dot{m}^S (y, 0) = \Xi (k^*) m^n + \eta m^S (0, 0) - (\delta + \varphi g (\varphi)) m^S (y, 0) \) \tag{E.1} 
\[ \dot{m}^R (0, N_i^R) = \delta m^R (y, N_i^R) - (\lambda_d + \eta) m^R (0, N_i^R) \] \tag{E.2} 
\[ \dot{m}^R (y, N_i^R) = (\Xi (k^{**}) - \Xi (k^*)) m^n + \eta m^R (0, N_i^R) - \delta m^R (y, N_i^R) \] \tag{E.3} 
\[ \dot{m}^B (0, N_i^B) = \delta m^B (y, N_i^B) - (\lambda_d + g (\varphi)) m^B (0, N_i^B) \] \tag{E.4} 
\[ \dot{m}^B (y, N_i^B) = (1 - \Xi (k^{**})) m^n + g (\varphi) m^B (0, N_i^B) - \delta m^B (y, N_i^B) \] \tag{E.5}

We focus on the steady state, where the measures of the different firm types do not change, i.e., \( \dot{m}^t (y, N_i^t) = 0 \). The above flow equations allow us to write the ratio of the steady state measures of type B firms \( m^B (0, N_i^B) \) to the measure of type S firms \( m^S (y, 0) \) as written in equation (11). Note, that the Inada conditions guarantee that the RHS of equation (11) increases in the innovation market tightness \( \varphi \) at a decreasing rate. Since in addition the RHS at \( \varphi = 0 \) exceeds the LHS, i.e., \( \text{RHS}(0) > 0 \), equation (11) determines the unique innovation market tightness \( \varphi \) for given innovation cost thresholds \( k^* \) and \( k^{**} \).

**Worker Flows and Labor Market Tightness:** We denote the measure of unemployed workers by \( u \). Unemployment evolves according to the difference between inflows and outflows, i.e.,

\[ \dot{u} = \begin{cases} 
\theta \lambda_m (\theta) u - \lambda_d (m^B (0, N_i^B) N_i^B + m^R (0, N_i^R) N_i^R) & \text{if } L_i^t = 0, \\
\theta \lambda_m (\theta) u - \delta (m^B (y, N_i^B) N_i^B + m^R (y, N_i^R) N_i^R) & \text{if } L_i^t = N_i^t. 
\end{cases} \] \tag{E.7}

Let us first consider the case when all firms keep their workers if they are hit by a productivity shock. We can determine the steady state measure of employed workers by equating the in- and outflow from unemployment, i.e.,

\[ \theta \lambda_m (\theta) u = \lambda_d (m^B (0, N_i^B) N_i^B + m^R (0, N_i^R) N_i^R), \]
\[ = \lambda_d \left( N_i^B + \frac{\Xi (k^{**}) - \Xi (k^*)}{1 - \Xi (k^{**})} N_i^R \right) m^B (0, N_i^B). \]

We denote the measure of employed workers by \( l \). The level of employment \( l \) can be obtained by summing over all type B and R firms, i.e.,

\[ l = (m^B (0, N_i^B) + m^B (y, N_i^B)) N_i^B + (m^R (0, N_i^R) + m^R (y, N_i^R)) N_i^R, \]
\[ = \left( \frac{\delta + \lambda_d + g (\varphi)}{\delta} N_i^B + \frac{\delta + \lambda_d + \eta \Xi (k^{**}) - \Xi (k^*)}{\delta} \frac{1}{1 - \Xi (k^{**})} N_i^R \right) m^B (0, N_i^B), \]
where the flow equations for firms in equations (E.1) to (E.6) imply,

$$\frac{1}{m^B (0, N_i^B)} = \left( \frac{1}{\varphi + \frac{\delta + \varphi g (\varphi)}{(\lambda_s + \eta) \varphi} + \frac{\delta + \lambda_d + g (\varphi)}{\delta}} \right) \frac{1}{m}$$

(E.8)

$$+ \frac{\delta + \lambda_d + \eta \Xi (k^{**}) - \Xi (k^*)}{\delta} \frac{1}{1 - \Xi (k^{**})}$$

Using the fact that the number of unemployed and employed workers have to add up to one, i.e., \(l = 1 - u\), allows us to write the labor market tightness \(\theta\) as a function of the number of workers employed at type \(B\) and \(R\) firms \(N_i^B\) and \(N_i^R\), as well as of \(\{\varphi, k^*, k^{**}, m\}\), i.e.,

$$\left( \frac{\lambda_d \delta + \theta \lambda_m (\theta)}{\delta} + \frac{\delta + g (\varphi)}{\delta} \right) N_i^B + \left( \frac{\lambda_d \delta + \theta \lambda_m (\theta)}{\delta} + \frac{\delta + \lambda_d + \eta \Xi (k^{**}) - \Xi (k^*)}{\delta} \right) \frac{\Xi (k^{**}) - \Xi (k^*)}{1 - \Xi (k^{**})} N_i^R \tag{E.9}$$

$$= \left( \frac{1}{\varphi + \frac{\delta + \varphi g (\varphi)}{(\lambda_s + \eta) \varphi} + \frac{\delta + \lambda_d + g (\varphi)}{\delta} + \frac{\delta + \lambda_d + \eta \Xi (k^{**}) - \Xi (k^*)}{\delta} \right) \frac{1}{m}$$

Let us now consider the case when all firms lay off workers if they are hit by a productivity shock. Note that \(N_i^R = N_i^B = N_i^L\). Equating in- and outflow into employment defines steady state employment as,

$$\frac{\theta \lambda_m (\theta)}{\delta} + \frac{\delta + g (\varphi) (\lambda_s + \eta) \varphi}{\delta} + \frac{\delta + \lambda_d + g (\varphi)}{\delta} + \frac{\delta + \lambda_d + \eta \Xi (k^{**}) - \Xi (k^*)}{\delta} \frac{1}{1 - \Xi (k^{**})} = \frac{m^B (0, N_i^B) N_i^L}{m}$$

where \(1/m^B (0, N_i^B)\) is given by equation (E.8). Substituting \(m^B (0, N_i^B)\) again implies,

$$\left( \frac{1}{\varphi + \frac{\delta + \varphi g (\varphi)}{(\lambda_s + \eta) \varphi} + \frac{\delta + \lambda_d + g (\varphi)}{\delta} + \frac{\delta + \lambda_d + \eta \Xi (k^{**}) - \Xi (k^*)}{\delta} \right) \frac{1}{m} \tag{E.10}$$

This again allows us to write the labor market tightness \(\theta\) as a function of the number of workers \(N_i^L\) employed at type \(B\) and \(R\) firms with \(y_i = y\), as well as \(\{\varphi, k^*, k^{**}, m\}\).

If only type \(B\) or only type \(R\) firms lay off workers, if they are hit by a productivity shock, steady state unemployment and the respective employment level are given by,

$$\theta \lambda_m (\theta) u = \begin{cases} \lambda_d m^B (0, N_i^B) N_i^B + \delta m^R (y, N_i^R) N_i^R \text{ if } L_i^B = 0 \text{ and } L_i^R = N_i^R, \\
\lambda_d m^R (0, N_i^R) N_i^R + \delta m^B (y, N_i^B) N_i^B \text{ if } L_i^B = N_i^B \text{ and } L_i^R = 0, \end{cases}$$

$$l = \begin{cases} (m^B (0, N_i^B) + m^B (y, N_i^B)) N_i^B + m^R (y, N_i^R) N_i^R \text{ if } L_i^B = 0 \text{ and } L_i^R = N_i^R, \\
m^B (y, N_i^B) N_i^B + (m^R (0, N_i^R) + m^R (y, N_i^R)) N_i^R \text{ if } L_i^B = N_i^B \text{ and } L_i^R = 0. \end{cases}$$

The flow equations (E.1) to (E.6) then determine the respective measures for the number of firms of type \(B\) and \(R\). Using the fact that all workers have to add up to one, i.e., \(l = 1 - u\),
allows us again to write the labor market tightness $\theta$ as a function of the number of workers employed at type $B$ and $R$ firms $N_i^B$ and $N_i^R$, as well as of $\{\varphi, k^*, k^{**}, m\}$, i.e., for $L_i^B = N_i^B$ and $L_i^R = 0$,

$$
\left(\frac{1}{\varphi} + \frac{\delta + \varphi g(\varphi)}{\varphi (\lambda_s + \eta)} + \frac{\delta + \lambda_d + g(\varphi)}{\delta} + \frac{\delta + \lambda_d + \eta \Xi (k^{**}) - \Xi (k^*)}{1 - \Xi (k^{**})}\right) \frac{1}{m} = \left(\frac{\lambda_d + g(\varphi)}{\theta \lambda_m (\theta)} + \frac{\lambda_d + g(\varphi)}{\delta}\right) N_i^B + \left(\frac{\lambda_d}{\theta \lambda_m (\theta)} + \frac{\delta + \lambda_d + \eta \Xi (k^{**}) - \Xi (k^*)}{1 - \Xi (k^{**})}\right) \frac{1}{m} \frac{\Xi (k^{**}) - \Xi (k^*)}{N_i^R},
$$

(E.11)

and for $L_i^B = 0$ and $L_i^R = N_i^R$,

$$
\left(\frac{1}{\varphi} + \frac{\delta + \varphi g(\varphi)}{\varphi (\lambda_s + \eta)} + \frac{\delta + \lambda_d + g(\varphi)}{\delta} + \frac{\delta + \lambda_d + \eta \Xi (k^{**}) - \Xi (k^*)}{1 - \Xi (k^{**})}\right) \frac{1}{m} = \left(\frac{\lambda_d}{\theta \lambda_m (\theta)} + \frac{\delta + \lambda_d + g(\varphi)}{\delta}\right) N_i^B + \left(\frac{\lambda_d}{\theta \lambda_m (\theta)} + \frac{\delta + \lambda_d + \eta \Xi (k^{**}) - \Xi (k^*)}{1 - \Xi (k^{**})}\right) \frac{1}{m} \frac{\Xi (k^{**}) - \Xi (k^*)}{N_i^R}.
$$

(E.12)

Keeping the variables $\{\varphi, k^*, k^{**}, m\}$ constant, equations (E.9) to (E.12) determine the respective increasing functions of the number of workers employed at the respective firms, i.e., $\theta (N_i^R, N_i^B)$ with $\partial \theta (N_i^R, N_i^B) / \partial N_i^I > 0$.

**Appendix F. Equilibrium**

An equilibrium is characterized by the market tightness in the innovation and the labor markets, the layoff decision of type $B$ and $R$ firms $L_i^B$ and $L_i^R$, the threshold values $k^*$ and $k^{**}$ of the innovation cost $k_i$ that determine the fraction of type $S$, $B$, and $R$ firms and the number of active firms in the economy $m$, i.e., by the set of variables $\{\varphi, \theta, L_i^B, L_i^R, k^*, k^{**}, m\}$.

The innovation market tightness $\varphi$ is determined by equation (11). Comparative statics using the implicit function theorem imply that innovation market tightness $\varphi$ decreases with both innovation cost thresholds $k^*$ and $k^{**}$, since, in the case of $k^*$, more firms decide to specialize in innovation and, in case of $k^{**}$, fewer firms decide to buy a new innovation when they are hit by a productivity shock. The layoff decision for firm types $B$ and $R$ are given by equations (4) and (5). They determine $L_i^R, L_i^B$ as a function of $\{\varphi, \theta\}$. Bargaining wages are given in Appendix Appendix B. The vacancy creation conditions are given in Appendix Appendix D. All vacancy creation curves define the number of employed workers as a decreasing function of labor market tightness, i.e., $N_i^I (\theta)$ with $\partial N_i^I (\theta) / \partial \theta < 0$. Substituting the respective functions $N_i^I (\theta)$ into the respective steady-state equations (E.9) to (E.12) determines labor market tightness as a function of $\{\varphi, L_i^B, L_i^R, k^*, k^{**}, m\}$. The property $\partial N_i^I (\theta) / \partial \theta < 0$ together with $\partial \theta (N_i^R, N_i^B) / \partial N_i^I > 0$, guarantees that the equilibrium market tightness is unique for a
given set of variables \( \{\phi, L^B_i, L^R_i, k^*, k^{**}, m\} \). The comparative static result that a higher number of firms \( m \) leads to higher labor market tightness \( \theta \) ensures that the free entry condition (10) is well defined.

The innovation cost thresholds \( k^* \) and \( k^{**} \) are determined by comparing the expected profits of the different types of firms as defined in equations (8) and (9). The single crossing property of the expected profits guarantees a unique pair of innovation cost thresholds \( k^* \) and \( k^{**} \) for a given set of variables \( \{\phi, \theta, m\} \). Thus, firms with low innovation costs specialize in innovation, firms with high innovation costs buy innovations when they are hit by a productivity shock and firms with medium innovation costs do own research if they are hit by a productivity shock.

The final equation that determines the number of firms \( m \) in equilibrium is the free entry condition (10), where the number of firms enters indirectly via the labor market tightness \( \theta \). A higher number of firms \( m \) increases, ceteris paribus, the labor market tightness \( \theta \). A higher labor market tightness increases the recruitment cost of workers and thus decreases the expected profit of type \( B \) and \( R \) firms. Thus, the free entry condition is decreasing in the number of firms.