Abstract

Convex vacancy creation costs shape firms’ responses to trade liberalization. They induce capacity constraints by increasing firms’ costs of production. A profit maximizing firm will therefore not fully meet the increased foreign demand, but serve only a few export markets. More productive firms will export to more countries and charge higher or similar prices compared to less productive firms. Trade liberalization also affects labor market outcomes. Increased profits by exporting firms trigger firm entry, reduce unemployment and increase wage dispersion in the on-the-job search model with monopolistic competition.

Keywords: On-the-job search; capacity constraints; international trade; heterogeneous firms; monopolistic competition

JEL-Codes: F16, F12, J64, L11


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1 Introduction

Four empirical observations of exporting firm behavior appear in the data:¹ i) Only part of all firms export. ii) Most of the exporting firms sell only to one foreign market and the frequency of firms that sell to multiple markets declines with the number of destinations. iii) Firms do not enter markets according to a common hierarchy. iv) The export strategy of one and the same firm varies widely across countries with similar characteristics.

The first empirical observation of exporting firm behavior can be explained by introducing firm heterogeneity into the Krugman (1980) trade model based on economies of scale in production and love-of-variety preferences as done by Melitz (2003). In order to explain the first three empirical observations Arkolakis (2010) and Eaton, Kortum, and Kramarz (2011) introduce not only market but also firm-specific heterogeneity in entry costs and market size. With respect to the fourth observation Eaton, Kortum, Kramarz (2011) note: “In particular, [our approach] leaves the vastly different performance of the same firm in different markets as a residual. Our analysis points to the need for further research into accounting for this variation.”

We provide an analytically tractable trade model that captures all four empirical regularities. We introduce capacity constraints into the new trade, monopolistic competition model with heterogeneous firms by Melitz (2003). As a result the size of exporting firms does not fully adjust in order to serve all foreign markets. Given the entry costs to each export market, firms rather react by selling only to a few markets at a higher price. Thus, even if only symmetric countries trade, exporting firms sell – depending on their productivity – only to part of the countries. Combined with the empirical observation of Pareto-distributed firm productivities (see for example Axtell, 2001), our model is able to explain all four empirical observations of exporting firm

¹See Dunne, Roberts and Samuelson (1989); Davis and Haltiwanger (1992); Bernard and Jensen (1995, 1999, 2004); Roberts and Tybout (1997); Clerides, Lach and Tybout (1998); Bartelsman and Doms (2000); Eaton, Kortum, and Kramarz (2004); Lawless (2009); and Eaton, Kortum, and Kramarz (2011).
behavior.

We assume that capacity constraints are caused by convex vacancy creation costs. While increasing marginal costs could stem for a variety of reasons, our predictions of the labor market effects after trade liberalization are well in line with empirical findings and anecdotal evidence of “labor shortage”. Firm heterogeneity implies that more productive firms or firms with a higher product quality will despite the convex costs of production hire more workers in order to be less capacity constrained. Convex vacancy creation costs make it attractive for firms to hire more workers by offering higher wages and attracting workers from other employers. In order to capture this recruiting channel we merge the Melitz (2003) with the on-the-job search model by Burdett and Mortensen (1998).

Given that our model successfully explains the main empirical facts of firm export-

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2 Magnier and Toujas-Bernate (1994) and Madden, Savage, and Thong (1994) argue that exporting firms may not always be able to meet the demands for its goods due to investment constraints. Ruhl and Willis (2008), Eaton, Eslava, Krizan, Kugler and Tybout (2009), and Fajgelbaum (2011) point out that firms need time to grow in order to be large enough to export. Redding and Venables (2004) and Fugazza (2004) find that country specific supply-side conditions can explain part of the differences in export performance. Manova (2008) isolates the effect of equity market liberalization on export behavior using panel data for 91 countries. Blum, Claro, and Horstmann (2010) assume that capital capacity constraints are responsible for fluctuations in export behavior of Chilean firms.

3 “Labor shortages” are often blamed for reducing firms’ ability to meet their demand. The ManpowerGroup provides extensive evidence of “labor shortage”, specifically of highly qualified workers in the “2011 Talent Shortage Survey” based on nearly 40,000 surveys of employers in 39 countries. There is also a heavy debate about the effects of “labor shortage” on the global competitiveness of China. The New-York-Times wrote on April 3, 2006 that “data from officials suggest that major export industries are looking for at least one million additional workers, and the real number could be much higher”. A Chinese supplier survey by Global Sources (2011) reports that “the persistent labor shortage has nearly driven growth in China’s export industries to a halt”. Lately The National Business Review wrote about the IT professional shortage in New Zealand (see http://www.nbr.co.nz/article/it-professional-shortage-continues-survey-118981) and Webmaster Europe, the International-European labor union for Internet professionals, stated that the IT professional shortage will continue in 2010 in Germany (see http://www.webmasters-europe.org/modules/news/article.php?storyid=95).
ing behavior, we investigate the effects of trade liberalization on the size of firms and the number of firms as well as on labor market outcomes. Trade liberalization leads to an increase of firms’ expected profits and triggers not only an increase of average firm size, but in contrast to Melitz (2003) also an increase in the number of firms. This is well in line with recent empirical findings by Eaton, Kortum, and Kramarz (2004, 2011) that suggest that a large fraction of the adjustment in market shares comes from changes in the number of firms and not from the adjustments of the amount sold by existing firms. At the same time, opening up to trade still forces less productive firms to leave the market like in Melitz (2003).

Additionally, trade liberalization increases wage dispersion since search frictions pin down the lowest wage at the level of unemployment benefits, while increased profits of exporting firms increase wages at the top of the distribution. Higher profits of exporting firms also increase job creation (both at the extensive and the intensive margin) and lead to less unemployment.

By developing a framework consistent with observed exporting behavior, we contribute to the literature that integrates imperfect labor markets into trade models and analyzes the effects of trade liberalization on unemployment and wage inequality. Brecher (1974) was the first to study minimum wages in the Heckscher-Ohlin model with two countries, two factors, and two goods, and Davis (1998) generalized this model. Davidson, Martin, and Matusz (1999) and Davidson and Matusz (2004) introduce search frictions and wage bargaining into multi-sector models of international trade governed by comparative advantage. More recently, Cuñat and Melitz (2007, 2010) study the effect of cross-country differences in firing restrictions on the patterns of comparative advantage in a Ricardian setting. Helpman, Itskhoki, and Redding (2009, 2010) allow firms to screen workers of different abilities in a Melitz

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4Evidence for increasing wage dispersion after trade liberalization is provided by Egger, Egger and Kreickemeier (2011) based on five European countries and by Helpman, Itskhoki, Muendler and Redding (2012) using linked employer-employee data for Brazil.

5This finding is well in line with recent empirical evidence that trade liberalization lowers unemployment provided by Dutt, Mitra, and Ranjan (2009) and Felbermayr, Prat and Schmerer (2011).
(2003) model with search and matching labor market frictions. Egger and Kreicke-meier (2012) explain intra-group wage inequality among ex ante identical workers due to a fair wage-effort mechanism. Amiti and Davis (2012) also assume a fair wage constraint, but focus on output tariffs. Using the Diamond-Mortensen-Pissarides matching model (see Pissarides, 2000) Felbermayr, Prat, and Schmerer (2011) show within a new trade theory model that unemployment falls if trade is liberalized. Fajgelbaum (2011) uses an on-the-job search equilibrium model based on Postel-Vinay and Robin (2002) to investigate how labor market frictions influence the growth path and export decision of firms. All of these papers explain part of the empirical findings discussed in the beginning, but none of them explains all four empirical facts.

The paper is structured as follows. In the next section we present the general framework that links the new trade model by Melitz (2003) with the on-the-job search model by Burdett and Mortensen (1998). In section 3 we analyze the equilibrium in a closed economy. In section 4 we investigate the effects of trade liberalization and compare the results with the literature focusing particularly on the comparison with Melitz (2003). Section 5 introduces vacancy creation with convex vacancy creation costs. We then provide a numerical example for the model with convex vacancy creation costs and show that our main effects prevail. Section 6 concludes and sets out future research objectives.

2 Framework

2.1 Labor market and workers’ search strategy

The model has an infinite horizon, is set in continuous time and concentrates on steady states. The measure of firms \( M \) in the economy will be endogenously determined in the product market. In the basic framework, we assume that all firms face the same fixed contact rate \( \eta \). This is identical to assuming that all firms open the same number of vacancies \( v \) due to zero vacancy creation costs for \( v \leq \bar{v} \) and infinitely high vacancy creation costs for \( v > \bar{v} \). In section 5 we allow firms with different productivities to
decide on the number of vacancies. In the basic model the total number of contacts made by active firms is given by \( \eta M \bar{v} \).

Workers’ life time is exponentially distributed with parameter \( \phi \). They are risk neutral and do not discount the future for simplicity but without loss of generality. Since we normalize the measure of workers to one, \( \phi \) also describes the in- and outflow of workers into the labor market. Workers are either unemployed and receive unemployment benefits \( z \), or they are employed. Both unemployed and employed workers are searching for a job with the same intensity. Following Burdett and Mortensen (1998) the probability of a worker to meet a firm follows a Poisson process with rate \( \lambda(M) \). Since aggregation requires that the total number of firm contacts equals the total number of worker contacts, the contact rate of a worker depends on the number of active firms \( M \) in the market, i.e.,

\[
\lambda (M) = \eta M \bar{v}.
\]

The contact rate of a worker is therefore increasing in the number of active firms in the economy.

Given the wage offer distribution \( F(w) \), a worker’s (flow) value \( \phi U \) of being unemployed equals unemployment benefits \( z \), plus the expected gain from searching. The latter depends on the contact rate \( \lambda(M) \) and the surplus from being employed rather than unemployed. Note, that the exit rate \( \phi \) acts as workers’ discount rate and the lowest and the highest wage paid in the economy are denoted by \( \underline{w} \) and \( \bar{w} \), respectively. Hence, \( \phi U \) is given by,

\[
\phi U = z + \lambda(M) \int_{\underline{w}}^{\bar{w}} \max \{ V(\bar{w}) - U, 0 \} dF(\bar{w}).
\]

(2)

The (flow) value of being employed \( \phi V(w) \) equals the current wage \( w \), plus the expected surplus from finding a better paid job, plus the expected loss from becoming unemployed, where \( \zeta \) denotes the exogenous job separation rate,

\[
\phi V(w) = w + \lambda(M) \int_{\underline{w}}^{\bar{w}} \max \{ V(\bar{w}) - V(w), 0 \} dF(\bar{w}) + \zeta [U - V(w)].
\]

(3)
As shown by Mortensen and Neumann (1988) the optimal search strategy for a worker is characterized by a reservation wage $w^r$, which is defined such that an unemployed worker is indifferent between accepting or rejecting a wage offer, i.e., $U = V(w^r)$. Using the above value functions it is straightforward to show that $w^r$ is independent of $F(w)$ and given by $w^r = z$. Thus, only wages that are at least as high as unemployment benefits $z$ are acceptable for unemployed workers. Furthermore, employed workers will only change employers if the wage $\tilde{w}$ offered by the outside firm exceeds the current wage $w$.

2.2 Product and new product ideas market

The consumers’ utility function is given by,

$$U = q_0 + Q,$$

where $q_0$ is an outside good serving as numéraire. Following Ethier (1982), Ludema (2002), Melitz (2003) and Helpman and Itskhoki (2010) the final output good $Q$ is a CES-aggregate, i.e.,

$$Q = \frac{1}{\rho} \left[ \int_{\omega \in \Omega} q(\omega)^\rho \, d\omega \right],$$

where $0 < \rho < 1$ and $\Omega$ equals the mass of available intermediate goods. Each intermediate good $\omega$ is produced by a single firm in a monopolistic competitive market. The mass of intermediate goods producers is equal to the number of active firms $M$ in the market. We assume perfect competition in the final goods market. Profit maximization of competitive final goods producers leads to the following demand for intermediate good $\omega$,

$$q(\omega) = p(\omega)^{-\frac{1}{1-\rho}}.$$

Labor $l(\omega)$ is the factor of production. As in Melitz (2003), firms differ in labor productivity such that the output of a firm that produces intermediate good $\omega$ is given by $q(\omega) = \varphi(\omega) l(\omega)$, where $\varphi(\omega)$ denotes the labor productivity of intermediate input.

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6 The numéraire good $q_0$ in the utility function in (4) absorbs all changes in aggregate demand.
producer $\omega$. As it is standard in the literature we use $\varphi$ to index intermediate input producers.

Intermediate input producers are risk neutral and live forever.\(^7\) Given the monopsony power that intermediate goods producers in the Burdett-Mortensen model have the size $l(\varphi)$ of the labor force employed by a firm depends on its wage $w$. Each firm is concerned about its steady state profit flow, which equals per period revenues $[\varphi l(\varphi, w(\varphi))]^\rho$ minus wage costs $w(\varphi)l(\varphi, w(\varphi))$ minus the per period fixed costs of production $f$. We assume that demand of an intermediate goods producer totally breaks down at the Poisson rate $\delta$. This reflects the end of the product cycle of a specific intermediate good.\(^8\) The rate $\delta$ acts as discount rate for firms. The steady state profit flow of an intermediate good producer with productivity $\varphi$ paying wages $w(\varphi)$ is given by,

$$\delta \Pi(\varphi, w(\varphi)) = [\varphi l(\varphi, w(\varphi))]^\rho - w(\varphi)l(\varphi, w(\varphi)) - f. \quad (7)$$

If demand for a specific intermediate good breaks down, a firm will acquire the patent of a new product at the market for new product ideas. The product ideas are sold in a perfectly competitive market. Existing intermediate good producers that compete for new product ideas differ in their stock of labor. The stock of workers $l(w(\varphi))$ that a firm employs depends in the Burdett-Mortensen model on the wage $w(\varphi)$ that the firm committed itself to pay to all its workers for their entire employment spell. A firm with labor force $l(w(\varphi))$ that pays the wage $w(\varphi)$ is thus willing to bid up to $\Pi(\varphi, w(\varphi))$ for a product idea $\varphi$. The product ideas $\varphi$ are drawn from a continuous distribution with c.d.f. $\Gamma(\varphi)$ and p.d.f $\gamma(\varphi)$ and support $(0, \overline{\varphi}]$.\(^9\) Denote by $\Pi(\varphi)$ the maximum

\(^7\)This standard assumption in the Burdett-Mortensen model allows us to refrain from considering the effects of firm growth.

\(^8\)Note, that we do not aim at modeling the life cycle of the product itself. We assume that firms have inherited a certain quality (or productivity) $\varphi$ in the past. See Vernon (1966) and Klepper (1996) for the idea of product life cycles.

\(^9\)We can think of $\varphi$ as labor productivity or as quality of a product idea. With the given form of product differentiation, these two interpretations are isomorphic (see Melitz, 2003, page 1699 and
discounted profits that a firm can make with a product idea $\varphi$, i.e.,

$$\Pi(\varphi) = \max_w \Pi(\varphi, w).$$

Then the firm with the labor force $l(w(\varphi))$, which happens, if it paid the wage $w(\varphi) = \arg \max_w \Pi(\varphi, w)$ in the past, will end up buying the product idea $\varphi$. Inventors of new product ideas have to invest in research and development at costs $f_e$ before they come up with a new product and get to know the quality of the idea. Since existing firms are only willing to buy profitable product ideas, only products with quality $\varphi \in [\varphi^*, \varphi]$ will be available in the market, where $\varphi^*$ is defined as the cutoff productivity, i.e.,

$$\Pi(\varphi^*) = 0. \tag{8}$$

Since all product ideas – except $\varphi^*$ – make positive profits, the expected discounted profits before knowing $\varphi$ are given by,

$$\Pi_e = [1 - \Gamma(\varphi^*)] \Pi = [1 - \Gamma(\varphi^*)] \int_{\varphi^*}^{\varphi} \Pi(\varphi) \frac{\gamma(\varphi)}{1 - \Gamma(\varphi^*)} d\varphi > 0,$$

where $1 - \Gamma(\varphi^*)$ equals the probability that an inventor will draw a $\varphi$ high enough to be profitable. $\Pi$ equals the average discounted profits of all product ideas available in the market.

Free entry of inventors ensures that new product ideas enter the market until the expected discounted profits before entering the market equal the fixed investment costs $f_e$, i.e.,

$$[1 - \Gamma(\varphi^*)] \Pi = \int_{\varphi^*}^{\varphi} \Pi(\varphi) \gamma(\varphi) d\varphi = f_e. \tag{9}$$

The zero cutoff profit condition (8) and the free entry condition (9) determine the number of active firms $M$ in the product market and the productivity $\varphi^*$ of the firm with the lowest productivity in the economy.

footnote 7). We use quality and productivity interchangeable, since intermediate good firms only care about profits per unit of labor.
2.3 Aggregation and steady state conditions

Aggregate profits are used to finance new product ideas and thus the initial research and development costs of inventors, i.e.,

\[ M \delta \bar{\Pi} = f_e I_e. \]  

(10)

\( I_e \) is the total mass of inventors that attempt entry and pay the fixed investment costs \( f_e \) each period. A large unbounded set of potential new product ideas ensures an unlimited supply of potential entrants into the market for new product ideas. Steady state requires that the flow into the pool of new product ideas is equal to the inflow of existing firms that want to buy new product ideas, i.e.,

\[ [1 - \Gamma (\varphi^*)] I_e = \delta M. \]  

(11)

It is straightforward to show that the steady state conditions (10) and (11) hold if the free entry condition (9) holds.

In steady state in- and outflows into and out of employment offset each other such that the distribution of employment over firms and the unemployment rate are stationary. Equating the flows in and out of unemployment gives the steady state measure of unemployed, i.e.,

\[ u = \frac{\kappa + \phi}{\kappa + \phi + \lambda (M)}. \]  

(12)

Note, that the aggregate number of matches in the economy increases and therefore the measure of unemployed decreases with the number of active firms \( M \) in the economy (due to equation (1)).

Equating the inflow and outflow into the group of workers earning less than \( w \) gives the steady-state wage earnings distribution \( G (w) \), i.e.,

\[ \lambda (M) F (w^-) u = G (w^-) (1 - u) \left[ \kappa + \phi + \lambda (M) \left[ 1 - F (w^-) \right] \right] \]  

\[ \Rightarrow G (w^-) = \frac{(\kappa + \phi) F (w^-)}{\kappa + \phi + \lambda (M) \left[ 1 - F (w^-) \right]}, \]  

(13)\( \quad \text{(14)}\)

where \( F (w^-) = F (w) - v(w) \), \( v(w) \) is the mass of firms offering a wage \( w \).
The steady state size of a firm $l(w)$ is determined by the hiring and quitting rate at a firm that pays wage $w$, i.e.,

$$l(w) = \frac{\eta \bar{v} [u + (1 - u) G(w^-)]}{\kappa + \phi + \lambda(M) [1 - F(w)]}.$$  

(15)

The number of recruited workers depends on the contact rate $\eta \bar{v}$ and on the probability that the contacted workers are willing to work for the wage $w$. If the wage $w$ exceeds the value of leisure $z$, then all unemployed workers $u$ and all workers employed at a lower wage will accept it, i.e., $(1 - u) G(w^-)$. The quitting rate is given by $\kappa$, the rate at which they quit into unemployment, by $\phi$, the rate at which workers exit the labor market, and by $\lambda(M) [1 - F(w)]$, the rate at which workers quit for a better paying job. Substituting $\lambda(M)$ using the aggregate matching condition (1), $u$ using the steady state measure of unemployed workers (12) and $G(w^-)$ using equation (14) allows us to write the steady state labor force of a firm that pays a wage $w$ as,

$$l(w) = \frac{\eta \bar{v} (\kappa + \phi)}{\kappa + \phi + \eta M \bar{v} [1 - F(w^-)] [\kappa + \phi + \eta M \bar{v} [1 - F(w)]]}. $$

(16)

Like in Burdett and Mortensen (1998) equation (16) implies that the size of a firm’s labor force $l(w)$ is increasing in the wage $w$, since firms with a high wage attract more employed workers and lose less workers to employers paying even higher wages. Equation (16) also shows that a higher number of active firms $M$ results in additional competition between firms and decreases the size of each firm’s labor force. As shown in section 5, this effect is still present but does not necessarily dominate if we endogenize the recruiting rate $\eta \bar{v}$, i.e., if we allow firms to increase their labor force by posting vacancies.

## 3 Equilibrium in a closed economy

### 3.1 Equilibrium definition

A steady state equilibrium is defined by the set $\{u, F(w(\varphi)), G(w(\varphi)), \varphi^*, M\}$, i.e., the unemployment rate, the wage offer and wage earnings distribution, the zero cutoff
productivity and the number of active firms in the economy. Firms choose a wage $w$ that maximizes the steady state profit flow of equation (7) given the wage offer distribution $F(w(\varphi))$, the number of active firms $M$ in the economy, the distribution $\Gamma(\varphi)$ and the cutoff productivity $\varphi^*$ as well as the optimal search strategy of workers. In the steady state equilibrium the unemployment rate $u$ and the wage earnings distribution $G(w(\varphi))$ in equations (12) and (14) have to be consistent with the wage offer distribution and steady state turnover of workers.

Inventors enter the market for new product ideas if the quality of their idea exceeds the cutoff productivity, i.e., $\varphi \geq \varphi^*$, where $\varphi^*$ is defined by equation (8). The number of active firms $M$ in the product market has to be such that the average profits of active firms are sufficiently large to finance the research and development costs $f_e$ of potential inventors such that the inflow of new product ideas equals the number of products whose demand breaks down. The respective steady state conditions in equations (10) and (11) or equivalently in the free entry condition (9) have to be satisfied in equilibrium.

### 3.2 Firms’ wage offers

Each firm chooses a wage $w(\varphi)$ that maximizes its steady state profit flow. As in Mortensen (1990) the posted wage weakly increases with $\varphi$. This can be seen from the equilibrium profit condition $\Pi(\varphi) = \max_w \Pi(\varphi, w)$ (and $w(\varphi) = \arg \max_w \Pi(\varphi, w)$) which implies,

\[
\delta \Pi(\varphi) = \varphi^\rho l(w(\varphi))^\rho - w(\varphi) l(w(\varphi)) - f,
\]

\[
\delta \Pi(\varphi) \geq \varphi^\rho l(w(\varphi'))^\rho - w(\varphi') l(w(\varphi')) - f.
\]

These equilibrium conditions imply for $\varphi > \varphi'$ that,

\[
\varphi^\rho l(w(\varphi))^\rho - w(\varphi) l(w(\varphi)) \geq \varphi^\rho l(w(\varphi'))^\rho - w(\varphi') l(w(\varphi'))
\]

\[
> (\varphi')^\rho l(w(\varphi'))^\rho - w(\varphi') l(w(\varphi'))
\]

\[
\geq (\varphi')^\rho l(w(\varphi))^\rho - w(\varphi) l(w(\varphi)).
\]
The difference of the first and the last term of these inequalities is greater than or equal to the difference of its middle terms, i.e.,

\[ \phi^\rho - (\phi')^\rho \leq \phi^\rho - (\phi')^\rho. \]

Since \( l(w(\varphi)) \) is an increasing function of the wage, it follows that wages \( w(\varphi) \) are weakly increasing in productivity \( \varphi \). The weak inequality already implies that the firm with the lowest productivity will pay no more than the reservation wage, i.e., \( w(\varphi^*) = z \).

In the basic model we focus on an equilibrium where wages strictly increase with productivity. In Appendix A we show that a mass point can only exist at the lower end of the wage distribution, which implies that firms at the lower end of the productivity distribution will create less vacancies. The respective equilibrium is therefore covered by the general case of endogenous vacancy creation in section 5. In Appendix A we also show that Assumption 1 is sufficient (although not necessary) to ensure that no mass point exists.

**Assumption 1:** In order to guarantee that no mass point exists, we assume,

\begin{equation}
\rho f > (1 - \rho) z \frac{\eta^\varphi (z + \phi)}{[z + \phi + \eta^\varphi]^2}.
\end{equation}

Given Assumption 1 wages \( w(\varphi) \) strictly increase with \( \varphi \) like in Mortensen (1990). Thus, the position of a firm in the wage offer distribution \( F(w(\varphi)) \) is equivalent to its position in the productivity distribution of active firms, i.e.,

\begin{equation}
F(w(\varphi)) = \frac{\Gamma(\varphi) - \Gamma(\varphi^*)}{1 - \Gamma(\varphi^*)} \text{ for all } \varphi \in [\varphi^*, \bar{\varphi}].
\end{equation}

The fact that a firm cannot adjust the size of its labor force freely to changes in output demand leads to a capacity constraint that implies that a firm will adjust its output price to changes in demand.

Since a firm can only recruit workers if it pays at least the reservation wage \( z \), the least productive firm that is active in the market will offer a wage equal to unemployment benefits \( z \). The optimal wage \( w(\varphi) \) posted by a firm with productivity \( \varphi > \varphi^* \)
is then given by,
\[
    w(\varphi) = \frac{1}{l(w(\varphi))} \left[\varphi l(w(\varphi))\right]^\rho - \int_{\varphi^*}^{\varphi} \frac{\rho}{\varphi} \left[\varphi l(w(\varphi))\right]^\rho d\varphi - f. \tag{19}
\]
The derivation can be found in Appendix B. Multiplying equation (19) by \(l(w(\varphi))\) reveals that total wage payments are given by revenues (the first term on the rhs in brackets) minus profits of a firm with productivity \(\varphi\) (the second term on the rhs in brackets) minus fixed costs.

### 3.3 Firm entry decision

Free entry of potential inventors ensures that the expected discounted profits earned in the product market \([1 - \Gamma(\varphi^*)] \bar{\Pi}\) is used to finance the fixed investment costs \(f_e\) as stated in equation (9). Substituting per period profits (7) and the optimal wage (19) implies the following free entry condition for inventors of product ideas,
\[
f_e = \frac{1}{\delta} \int_{\varphi^*}^{\varphi} \left[\int_{\varphi^*}^{\varphi} \frac{\rho}{\varphi} \left[\varphi l(w(\varphi))\right]^\rho d\varphi\right] \gamma(\varphi) d\varphi. \tag{20}
\]
The expected discounted profits decrease with the number of active intermediate good producers, because the size of a firm’s labor force \(l(w(\varphi))\) is a decreasing function of the number of active firms \(M\). At the same time the expected discounted profits of an inventor increase if the cutoff productivity decreases, because the likelihood of having a productivity draw that can be soled to an intermediate goods producer increases. Using the implicit function theorem, we show in Appendix C that the free entry condition defines a decreasing relation between the zero cutoff productivity \(\varphi^*\) and the number of active intermediate goods producers \(M\) in the market.

An intermediate goods producer has to offer at least the level of unemployment benefits \(z\) in order to attract any worker. Given this lower bound of the support of the wage offer distribution \(F(w(\varphi))\), the zero cutoff productivity firm \(\varphi^*\) employs \(l(z) = l(w(\varphi^*))\) workers. Utilizing the per period profits definition (7) implies that the zero cutoff productivity level \(\varphi^*\) is given by,
\[
\left[\varphi^* \frac{\eta \bar{v} (\varphi + \phi)}{[\varphi + \phi + \eta M \bar{\eta}]^2}\right]^\rho = z \frac{\eta \bar{v} (\varphi + \phi)}{[\varphi + \phi + \eta M \bar{\eta}]^2} + f. \tag{21}
\]
Since the zero cutoff productivity firm pays the reservation wage $z$, it will only attract unemployed workers and lose its workers to all other firms that pay higher wages. Consequently, a higher number of active firms $M$ increases the number of quits at the zero cutoff productivity firm and, therefore, reduces its steady state labor input. This decreases the firm’s net revenue. The firm will subsequently no longer be able to cover the wage payments and the fixed costs $f$. Thus, only more profitable firms will be able to survive in the market, which increases the zero cutoff productivity. Using the implicit function theorem and Assumption 1 we show in Appendix C that the zero profit condition defines an increasing relation between the zero cutoff productivity $\varphi^*$ and the number of active firms $M$ in the market. Thus, the free entry condition and the zero cutoff condition determine a unique equilibrium as shown in Figure 1, as long as unemployment benefits $z$ and the per period fixed costs $f$ are low enough to ensure that an equilibrium exists.

Figure 1: Number of active intermediate goods producers and cutoff productivity
4 Open economy

Assume that there are \( n + 1 \) identical countries that differ only in the variety \( \Omega \) of goods that they produce. Given that final output producers love variety, they are interested in trading with other countries. Due to the symmetry of countries, intermediate goods producers face the same demand curve in each export market as they face in the domestic market, i.e., \( q(\varphi) = p(\varphi)^{1/(\rho-1)} \). Serving an export market involves some fixed costs \( f_x \geq f \) per period and some proportional shipping costs per unit shipped. Thus, the price of an export good at the factory gate is given by \( p_x(\varphi) / \tau = p_d(\varphi) \), where \( p_d(\varphi) \) denotes the price in the domestic market.

Given that an exporting firm with productivity \( \varphi \) can only produce the fixed output \( l(w(\varphi)) \), it will choose the number of export markets \( j \) such that the output sold in \( j \) export markets and the domestic market maximizes profits. Serving all export markets is not profit-maximizing given the capacity constraints and the exporting fixed costs. Thus, a firm that decided to serve a subset \( j \leq n \) of foreign markets maximizes its profits if it sells

\[
q_d(\varphi) = \frac{1}{1 + j\frac{\tau^{\frac{\rho}{\rho-1}}}{\rho-1}} q(\varphi), \quad \text{and} \tag{22}
\]

\[
q_x(\varphi) = \frac{\tau^{\frac{\rho}{\rho-1}}}{1 + j\frac{\tau^{\frac{\rho}{\rho-1}}}{\rho-1}} q(\varphi), \tag{23}
\]

at the domestic and at each export market, respectively (see derivation in Appendix D). The profits of a firm serving \( j \) export markets are therefore given by,

\[
\delta\Pi_{d+j}(\varphi) = \max_{w} \left[ \left( 1 + j\frac{\tau^{\frac{\rho}{\rho-1}}}{\rho-1} \right)^{(1-\rho)} \left[ \varphi l(\varphi) \right]^\rho - w(\varphi) l(w(\varphi)) - f - jf_x \right]. \tag{24}
\]

In addition to the closed economy setting a firm with productivity \( \varphi \) decides not only on the wage \( w(\varphi) \) but also on the number \( j \) of countries it wants to export to. Hence, it will choose the number of export markets such that profits are maximized, i.e., \( \Pi_{\text{max}}(\varphi) = \max_j \Pi_{d+j}(\varphi) \).
4.1 Trade pattern

Denote by $\varphi^j_x$ the export cutoff productivity for a firm that decides to export to $j \leq n$ countries. Firms with $\varphi \geq \varphi^j_x$ find it optimal to export to $j$ or more countries while firms with $\varphi < \varphi^j_x$ will only serve less than $j$ foreign markets and the domestic market (or only the domestic market). Wages chosen by firms have to satisfy the first order condition like in a closed economy. The non-exporting firm with the lowest productivity level $\varphi^*$ will pay the reservation wage $z$ such that unemployed workers are willing to start working. As shown in Appendix D the wage equation $w(\varphi)$ for exporting firms is given by,

$$w(\varphi) = \frac{1}{l(w(\varphi))} \left[ 1 + j \tau \rho^{\varphi-1} \right]^{(1-\rho)} \left[ \varphi l(w(\varphi)) \right]^\rho - f - j f_x$$

$$- \frac{1}{l(w(\varphi))} \sum_{i=1}^{j+1} \left[ 1 + (i - 1) \tau \rho^{\varphi-1} \right]^{(1-\rho)} \int_{\varphi_i-1}^{\varphi_i} \frac{\rho}{\varphi} \left[ \varphi l(w(\varphi)) \right]^\rho d\varphi,$$

where $\varphi_{x}^{j+1} = \varphi$ and $\varphi_{x}^0 = \varphi^*$.

Note that profit maximization ensures that the wage function does not jump upward at $\varphi^j_x$, i.e., that the support of the wage distribution is connected. To see this suppose the opposite, i.e., that the exporting firm with the lowest productivity $\varphi^j_x$ were to pay a wage $w(\varphi^j_x) = w(\varphi) + \Delta$, where $\Delta > 0$ denotes the jump at $w(\varphi)$ where productivity is given by $\varphi = \varphi^j_x - \varepsilon$ for any small $\varepsilon > 0$. The wage jump does not increase the number of workers of firm $\varphi^j_x$ since it has the same position in the wage distribution as before. It is, therefore, optimal for the firm to pay a wage that is only slightly above $w(\varphi)$ and save the wage costs $\Delta$ per worker. Thus, the wage function is continuous on $[\varphi^*, \bar{\varphi}]$.

Firms with a low productivity serve only the domestic market, while firms with a high productivity will also export. The firm with the export cutoff productivity $\varphi^j_x$ is indifferent between serving $j$ export markets and the domestic market or serving $j-1$ export markets and the domestic market, i.e., $\delta \Pi_{d+j}(\varphi^j_x) = \delta \Pi_{d+j-1}(\varphi^j_x)$. As proven in Appendix D more productive firms will export to more countries. Specifically, the
export cutoff productivity is given by,

\[
\phi_j x l^\rho = \frac{f_x}{[1 + j \tau / (\rho - 1)](1 - \rho) - [1 + (j - 1) \tau / (\rho - 1)](1 - \rho)}.
\]  
(26)

**Proposition 1** The number of export markets \( j \leq n \) served by a firm is increasing in its productivity, i.e., the export cutoff productivity \( \phi_j^l \) is increasing in \( j \).

**Proof.** See Appendix D. ■

Proposition 1 implies that more productive firms will serve more markets, and that an exporting firm may not serve all markets, even if the destinations are very similar. These predictions are well in line with recent empirical findings by Lawless (2009) and by Eaton, Kortum, and Kramarz (2011) that firms do not enter export markets according to a common hierarchy depending on export destination characteristics. Our theory therefore nicely complements the explanation based on market as well as firm-specific heterogeneity in entry costs or market size given by Arkolakis (2010) and Eaton, Kortum, and Kramarz (2011). In addition Proposition 1 is able to explain why the export strategy of one and the same firm varies widely across countries with similar characteristics.

Our explanation of these trade patterns is based on capacity constraining labor market frictions, which implies – all else equal – that firms that recruit their workers in more flexible labor markets will serve more export markets. In order to empirically identify the link between the capacity constraining effect of labor market frictions and exports, one needs variation in labor market frictions over time and across countries that does not coincide with other trade liberalization policies. Testing the predictions of our model therefore requires a very rich data-set with export destinations given on the firm-level for a large set of countries. This country-panel then has to be linked to labor market indices that characterize labor market frictions.\(^{10}\) Once such data becomes available it should be straightforward to test the predictions of our model.

\(^{10}\)The dataset that comes closest to fulfill these requirements is the “EFIGE - European Firms in a Global Economy” data-set for European countries, which was compiled with an enormous amount of effort by Haltiwanger, Scarpetta and Schweiger (2010) for 16 developed and emerging economies (see
4.2 Firm structure and prices

Given the export decisions of firms with different productivities, we are able to determine the expected profits of active firms in an economy \( [1 - \Gamma (\varphi^*)] \Pi \). The free entry condition requires that expected profits equal the fixed investment costs \( f_e \), i.e.,

\[
f_e = \frac{1}{\delta} \int \frac{\tilde{\varphi}}{\varphi^*} \left[ \sum_{i=1}^{j+1} \left[ 1 + (i - 1) \frac{\varphi}{\varphi^*} \right] (1 - \rho) \int_{\varphi_{i-1}}^{\varphi_i} \frac{\rho}{\tilde{\varphi}} \left[ \tilde{\varphi} l (w (\tilde{\varphi})) \right] d\tilde{\varphi} \right] \gamma (\varphi) d\varphi,
\]

(27)

where \( \varphi_{j+1} = \varphi \) and \( \varphi_0 = \varphi^* \). The derivation of equation (27) is given in Appendix D. Since average profits increase due to trade, the free entry curve in Figure 2 rotates outward if trade is liberalized. The zero cutoff condition (21) remains unchanged if trade is liberalized, since the firm with the lowest productivity will pay the reservation wage \( z \) and only sells at the domestic market (compare Figure 2).

The higher expected profits in an open economy compared to a closed economy trigger entry and increases the number of active firms \( M \) for a given cutoff productivity \( \varphi^* \). Given the increased number of active firms in the economy, potential entrants realize that their labor force will be lower than in the closed economy and that they will not be able to produce enough to pay the per period fixed costs \( f \). Thus, the zero cutoff productivity increases and the product ideas with the lowest productivity do not enter the market for new products.

**Proposition 2** Given Assumption 1, the zero cutoff productivity \( \varphi^* \) and the number of active firms \( M \) in an open economy is higher than in autarky. The size of all firms \( l (w (\varphi)) \) decreases.

**Proof.** See Appendix D. ■

The monopsonistic labor market changes firms reactions to trade liberalization compared to the reaction of firms in a frictionless labor market like in Melitz (2003). In

for more details at [http://www.efige.org/](http://www.efige.org/). However, even this data-set would have to be enriched by detailed firm information, specifically, about whether a firm exports or not, and if, in which countries it exports.
a perfect labor market exporting firms increase labor input until their marginal product is reduced to equal the market wage. The higher demand for labor by exporting firms is meet at the costs of a lower labor input at non-exporting firms. In a frictional labor market without vacancy creation the size of a firm’s labor force is determined by the position of a firm in the wage offer distribution. Thus, exporting firms are not able to increase their output since their labor input is given by their position in the wage distribution. The position of exporting firms in the wage distribution decreases because the cutoff productivity increases. Hence, opening up to trade will decrease a firm’s labor force in the new steady-state. In addition it triggers entry of new firms that compete for the same number of workers reducing the number of workers per firm even further. In contrast to Melitz (2003), trade therefore increases the number of active firms and does not lead to bigger firms. Figure 3 shows the firm size reactions in a frictional labor market and compares it to the perfect labor market environment of Melitz (2003). This result is specific to the simple case of no vacancy creation. In section 5, where we allow for vacancy creation, exporting firms with a high productivity will be larger while firms with a low productivity will be smaller compared to autarky.
Since exporting firms find it very costly to increase their output in response to the increase in foreign demand, they respond to the increased demand by increasing their prices. The prices charged by exporting firms in the domestic market are no longer lower for exporting firms compared to domestic firms like in Melitz (2003). As Figure 4 suggests, they are in the same range as the prices of domestic firms. The exact relation depends on the quantities of output sold as stated in the following Proposition.

**Proposition 3** Given Assumption 1, the highest domestic price of firms that export to $0 \leq j \leq n$ countries is higher than the highest price of firms exporting to $j - 1$ countries if and only if

\[
\frac{\varphi_x^{j-1} I(w(\varphi_x^{j-1}))}{1 + (j - 1) \tau^{\beta/(\rho-1)}} > \frac{\varphi_x^j I(w(\varphi_x^j))}{1 + j \tau^{\beta/(\rho-1)}},
\]

where $\varphi_x^0 = \varphi^*$ and $w(\varphi_x^0) = z$.

**Proof.** The firm that charges the highest price of all firms exporting to $j$ countries is the firm with export cutoff productivity $\varphi_x^j$. It produces and sells the smallest quantity of the good and therefore charges the highest price of all firms exporting to $j$ countries, i.e., $\varphi_x^j = \arg \max_{\varphi \in [\varphi_x^j, \varphi_x^{j+1}]} p_d(\varphi)$. Due to the downward sloping demand functions we know that $p_d(\varphi_x^j) < p_d(\varphi_x^{j-1})$ holds if and only if $q_d(\varphi_x^j) > q_d(\varphi_x^{j-1})$. □
Figure 4: Prices in autarky and in an open economy

While prices decrease with productivities in a perfect labor market, in a frictional environment exporting firms charge similar prices compared to domestic firms because capacity constraining labor market frictions induce exporting firms to maximize their profits by selling only a limited quantity per market. These price patterns are well supported by the empirical findings of Bughin (1996) and De Loecker and Warzynski (2009). Bughin (1996) finds that the markup charged by firms increases with capacity constraints and boosts export prices and De Loecker and Warzynski (2009) find that exporters charge on average higher markups and that markups increase upon export entry.

Another explanation for similar domestic prices of non-exporting and exporting firms that the empirical literature has suggested is the higher quality of the goods produced by exporting firms (compare Fajgelbaum, 2011; and Kugler and Verhoogen, 2012). This explanation is also consistent with our model.

4.3 Unemployment and wages dispersion

Opening up to trade increases expected profits, triggers firm entry and reduces unemployment. Like in Felbermayr, Prat, and Schmerer (2011), Helpman and Itskhoki
and Helpman, Itskhoki, and Redding (2009, 2010) additional demand from abroad increases firms’ revenue and their demand for labor. While firms in Felbermayr, Prat, and Schmerer (2011), Helpman and Itskhoki (2010) and Helpman, Itskhoki, and Redding (2009, 2010) create additional vacancies in order to increase labor input, in our frictional environment additional firms enter the market since the simple framework does not allow them to increase their recruitment rate by opening new vacancies.

In the given context opening up to trade still implies that the firm with the lowest productivity will pay the reservation wage $z$ like in autarky. Since the zero cutoff productivity increases compared to autarky, i.e., $\varphi^*_T > \varphi^*_A$, some firms with a productivity $\varphi \geq \varphi^*_T > \varphi^*_A$ that paid a wage above $z$ will decrease their wages since they now occupy a lower position within the wage offer distribution. If we hold the position of a firm in the wage distribution constant, trade liberalization increases wages because the marginal revenue of all firms increases due to the lower number of employees that they are able to recruit. Exporting firms experience even a higher increase in their marginal revenues since they can now charge higher prices by serving not only the domestic but also foreign markets. Thus, two counteracting effects drive wage changes: (i) the positive effect of an increase in the marginal revenue of a firm and, (ii) the negative effect of a lower position in the wage distribution. Of course, the negative effect is zero for the firm with the highest productivity $\overline{\varphi}$, such that wages increase at the upper end of the wage distribution. Since wages at the bottom of the wage distribution are held constant by the level of unemployment benefits $z$, it follows that the dispersion of wages measured as difference between the highest and the lowest wage is higher in an open economy than in autarky. The effect on the average wage is ambiguous and depends on the shape of the productivity distribution as well as the job finding and job destruction rate that translate the wage offer distribution into the wage earnings distribution as stated in equation (14). Proposition 4 summarizes the effect of trade liberalization on unemployment and wage dispersion.

**Proposition 4** Given Assumption 1, opening up to trade reduces the unemployment rate $u$ and increases wage dispersion, i.e., increases $w(\overline{\varphi}) - z$, compared to autarky.
Proof. See Appendix E. ■

Figure 5: Wages in autarky and in an open economy

The results concerning the effects of trade liberalization on the wage distribution of ex-ante identical workers are similar to the findings of Amiti and Davis (2012), Egger and Kreickemeier (2012), and Helpman, Itskhoki, and Redding (2009, 2010). However, in our context wage inequality is not the result of exogenously given fair-wage preferences\footnote{Whereas in Egger and Kreickemeier (2009) fair-wage preferences are linked to productivity differences between firms, they are based on profits of firms in Amiti and Davis (2012) and Egger and Kreickemeier (2012).} or the result of monitoring or screening costs, but rather the result of continuous search for better jobs of workers, as introduced in Burdett and Mortensen (1998).
5 Vacancy creation in an open economy

5.1 The matching technology

In previous sections the analysis was based on the assumption that all firms have a constant recruitment rate $\eta \bar{v}$ and cannot expand their production by opening new vacancies in response to an increase in foreign demand. In this section we allow firms to influence their contact rate by posting vacancies like in Mortensen (2003). The contact rate of a firm with productivity $\phi$ depends on the number of vacancies $v(\phi)$ it posts. The total number of contacts in an economy (and the contact rate of workers) is therefore given by,

$$\lambda(M \tilde{v}) = \eta M \int_{\phi^*}^{\phi} \frac{v(\phi)}{1 - \Gamma(\phi^*)} d\Gamma(\phi) = \eta M \tilde{v}. \tag{28}$$

The per period costs of vacancy creation is an increasing function of the vacancies opened, i.e., $c(v) = \xi v(\phi)^\alpha$. This cost function allows us to compare our results with the case of constant vacancy creation costs, $\alpha = 1$, like in Felbermayr, Prat and Schmerer (2011), who link the Diamond-Mortensen-Pissarides model (see Pissarides, 2000) with the Melitz (2003) model.

5.2 Labor market and trade pattern

In an open economy a firm with productivity $\phi$ chooses its wage $w(\phi)$ and its number of vacancies $v(\phi)$ such that per period profits are maximized for a given number of export markets $j$, i.e.,

$$\delta \Pi_{d+j}(\phi) = \max_{w,v} \left[ 1 + j \tau \frac{\phi}{\rho} \right]^{(1-\rho)} [\phi l(\phi, v)]^\rho - w(\phi) l(\phi, v) - \frac{c}{\alpha} v^\alpha - f - jf_x \right]$$

s.t. $l(\phi, v) = \frac{\eta v(\phi + \phi)}{[\phi + \phi + \lambda(M \tilde{v}) [1 - F(w(\phi))]]^2}$. \tag{29}$

The number of employees $l(\phi, v)$ working for a firm with productivity $\phi$ increases proportionally with the number of vacancies like in Mortensen (2003) and with the wage like in Burdett and Mortensen (1998). Thus, firms can increase their labor input by increasing their wage and by opening more vacancies.
Firms with a higher productivity will always pay higher wages. Taking this into account and the fact that the contact rate is proportional to the number of vacancies posted by a firm, the wage offer distribution is the vacancy weighted distribution of productivities, i.e.,

\[ F(w(\varphi)) = \frac{\int_{\varphi^*}^{\varphi} v(\varphi) \, d\varphi}{\int_{\varphi^*}^{\varphi} v(\varphi) \, d\varphi} \, \Gamma(\varphi). \tag{30} \]

Firms choose wages such that the resulting increase in labor balances marginal revenue with marginal labor costs. The number of vacancies are chosen such that the marginal net revenue generated by the last opened vacancy equals the marginal costs of creating the vacancy. The optimality conditions for wages and vacancies are therefore given by,

\[ \begin{align*}
\rho \left[ 1 + j \tau^{\rho-1} \right] & \varphi^\rho l(\varphi, v)^{(\rho-1)} - w(\varphi) \frac{\partial l(\varphi, v)}{\partial v} = cv^{\alpha-1}, \\
\rho \left[ 1 + j \tau^{\rho-1} \right] & \varphi^\rho l(\varphi, v)^{(\rho-1)} - w(\varphi) \frac{\partial l(\varphi, v)}{\partial \varphi} = l(\varphi, v) \frac{\partial w(\varphi)}{\partial \varphi},
\end{align*} \tag{31} \tag{32} \]

where the differential equation (32) follows from the fact that more productive firms pay higher wages. Substituting \( F(w(\varphi)) \) according to equation (30) and (28) in equation (29) yields,

\[ l(\varphi, v) = \frac{\eta v(\varphi + \varphi^*)}{\left[ \varphi + \varphi^* + \eta M \int_{\varphi^*}^{\varphi} \frac{v(\varphi)}{1 - \Gamma(\varphi^*)} \, d\varphi \right]^2}. \tag{33} \]

Inserting into the first order conditions implies the following first order differential wage equation,

\[ \frac{\partial w(\varphi)}{\partial \varphi} = \rho \left[ 1 + j \tau^{\rho-1} \right] \varphi^\rho l(\varphi, v)^{(\rho-1)} - w(\varphi) \frac{1}{l(\varphi, v)} \frac{\partial l(\varphi, v)}{\partial \varphi}, \tag{34} \]

with the terminal condition \( w(\varphi^*) = z \).

The number of vacancies created by the firm is implicitly defined by the vacancy creation condition (31), where the wage \( w(\varphi) \) is given by the solution to the differential equation (34). The average number of vacancies \( \tilde{v} \) per active firm is obtained by integrating the vacancies created by active firms, i.e.,

\[ \tilde{v} = \int_{\varphi^*}^{\varphi} \frac{v(\varphi)}{1 - \Gamma(\varphi^*)} \, d\varphi. \tag{35} \]
The number of export countries a firm is willing to enter depends – like in the simple model – on the comparison of profits from exporting to \( j \) or \( j - 1 \) countries, i.e.,

\[
\Pi_{d+j} (\varphi) \geq \Pi_{d+j-1} (\varphi).
\] (36)

### 5.3 Product market

The product market equilibrium is defined by two conditions, the free entry condition that determines the number of active firms in the economy \( M \) given the vacancy creation decision in the labor market (that determines the average number of vacancies \( \bar{v} \) per active firm) and the zero cutoff productivity condition that determines the productivity level \( \varphi^* \) that guarantees non-negative profits.

Firms only acquire a product idea if the profits they are able to generate are positive. Using the vacancy creation condition (31) one can write profits of a firm serving the home and \( j \) export markets as,

\[
\delta \Pi_{d+j} (\varphi) = (1 - \rho) \left[ 1 + j \tau \varphi \right]^{1-\rho} \left[ \varphi^* l (\varphi) \right]^\rho + \left( 1 - \frac{1}{\alpha} \right) cv (\varphi)^\alpha - f - j f_x. \] (37)

Since the firm with the lowest productivity pays a wage equal to unemployment benefits, the zero cutoff productivity \( \varphi^* \), defined as \( \delta \Pi_d (\varphi^*) = 0 \), is given by the solution to the system of two equations determining the zero cutoff productivity \( \varphi^* \) and the number of vacancies \( v (\varphi^*) \) created by the cutoff productivity firm, i.e.,

\[
(1 - \rho) [\varphi^* l (\varphi^*)]^\rho + \left( 1 - \frac{1}{\alpha} \right) cv (\varphi^*)^\alpha = f, \] (38)

\[
\rho [\varphi^* l (\varphi^*)]^\rho - z l (\varphi^*) = cv (\varphi^*)^\alpha. \] (39)

The labor force size of the zero cutoff productivity firm is according to equation (29) given by,

\[
l (\varphi^*) = \frac{\eta v (\varphi^*) (\varphi + \phi)}{[\varphi + \phi + \eta M \bar{v}]^2}.
\]

The free entry condition ensures that the profits generated by all firms are used to pay the investment costs \( f_e \) of inventors of new product ideas. The expected discounted
profits of exporting and non-exporting firms can be written as follows,

\[ f_e = \int_{\varphi^*}^{\bar{\varphi}} \Pi_{\max} (\varphi) \gamma (\varphi) \, d\varphi, \]

(40)

where \( \Pi_{\max} (\varphi) = \max_i \Pi_{d+i} (\varphi) \) denotes the maximum profits attainable by an intermediate goods producer with productivity \( \varphi \).

5.4 The case of linear vacancy creation costs (\( \alpha = 1 \))

If vacancy creation costs are linear, i.e., \( \alpha = 1 \), the vacancy creation condition reveals that firms increase their number of vacancies up to the point where marginal revenues are equalized across productivities (derivation is given in Appendix F), i.e.,

\[ \rho \left[ 1 + j \tau \frac{\rho}{\rho - 1} \right]^{(1-\rho)} \varphi^d (\varphi, j)^{(\rho-1)} = c \frac{v (\varphi^*)}{l (\varphi^*)} + z. \]

(41)

Firms choose the number of export markets \( j \) such that profits are maximized, i.e., \( \max_j \Pi_{d+j} (\varphi) \). Since marginal revenues are equalized across firms, all exporting firms increase their production in order to serve all export markets.

**Proposition 5** If vacancy creation costs are linear, then all exporting firms serve all \( n \) foreign markets. The unique export cutoff is given by,

\[ \varphi_x = \frac{\tau}{\rho} \left[ \frac{f_x}{(1-\rho)} \right]^{1-\rho} \left[ c \frac{v (\varphi^*)}{l (\varphi^*)} + z \right]. \]

(42)

**Proof.** Taking the profit equation (37) for \( \alpha = 1 \) and substituting \( [\varphi l (\varphi, j)]^\rho \) using equation (41) gives,

\[ \delta \Pi_{d+j} (\varphi) = (1 - \rho) \left[ 1 + j \tau \frac{\rho}{\rho - 1} \right] [\rho \varphi]^{\rho/\rho} \left[ c \frac{v (\varphi^*)}{l (\varphi^*)} + z \right]^{-\rho/\rho} - f - j f_x. \]

The export cutoff productivity is given by equating the profits of exporting to \( j \) or \( j - 1 \) countries, i.e., \( \delta \Pi_{d+j} (\varphi^*_x) = \delta \Pi_{d+j-1} (\varphi^*_x) \). Substituting profits from the equation above gives the export cutoff in equation (42). Thus, the export cutoff is the same for any number \( j \) of export destinations. Differentiating the profit equation with respect
to $\varphi$ shows that for $\varphi < \varphi_x$ marginal profits are negative for any $j > 0$ and for $\varphi \geq \varphi_x$ marginal profits are positive for any $j > 0$. Thus, the export cutoff is unique. □

Thus, with linear vacancy creation costs exporting firms create so many vacancies that their output is large enough to meet the additional demand of all $n$ export countries like in Felbermayr, Prat, Schmerer (2011), Helpman and Itskohki (2010) and Helpman, Itskohki, and Redding (2009, 2010).

Wages are still dispersed although marginal revenues are constant across productivities (derivation is given in Appendix F), i.e.,

$$w(\varphi) = c \left[ \frac{v(\varphi^*)}{l(\varphi^*)} - \frac{v(\varphi)}{l(\varphi)} \right] + z.$$

The reason is the same as in the simple Burdett-Mortensen model. If firms paid the same wage, each firm would have an incentive to deviate and offer a slightly higher wage since it will then be able to recruit also workers employed at other firms and would therefore be able to recruit additional workers at no extra costs (i.e., could save on vacancy creation costs). Thus, in equilibrium firms with high productivities pay high wages and have low turnovers, while firms with low productivities pay low wages and have high turnovers.

5.5 The case of convex vacancy creation costs ($\alpha > 1$)

As shown by the following simulation, in the case of convex vacancy creation costs and on-the-job search labor market frictions all our propositions hold with one exception: Exporting firms with sufficiently high productivity will be larger in a global economy compared to autarky.

5.5.1 Simulation method

As the model with endogenous vacancy creation can no longer be solved analytically, we rely on numerical solutions. We assume productivity to be Pareto distributed,

$$\Gamma(\varphi) = \frac{[\varphi]^{-\gamma} - \varphi^{-\gamma}}{[\varphi]^{-\gamma} - [\varphi^*]^{-\gamma}}$$

and

$$\gamma(\varphi) = \frac{\gamma \varphi^{-\gamma-1}}{[\varphi]^{-\gamma} - [\varphi^*]^{-\gamma}}.$$
In order to simulate the model, we proceed as follows. First we construct a grid of \( \varphi \), running from \( \varphi \) to \( \bar{\varphi} \) in equal steps. Afterwards we specify a starting value for \( \varphi^* \) somewhere above \( \varphi \) and below \( \bar{\varphi} \). For each element of the vector \( \varphi \) we check whether the value of \( \varphi \) is greater than \( \varphi^* \). If not, we assign the value zero to the vector of \( \varphi \).

Next, we multiply the values of this vector with the step size of \( \varphi \) and initialize the vector for the vacancies \( v(\varphi) \). Later in subsequent loops the vacancies \( v(\varphi) \) are determined according to equation (31) given wages \( w(\varphi) \) and labor inputs \( l(\varphi) \). We then construct a vector that contains the integral \( \int_{\varphi}^{\bar{\varphi}} \frac{v(\widetilde{\varphi})}{1-F(\varphi^*)} d\Gamma(\widetilde{\varphi}) = \tilde{v} \) for each value of \( \varphi \).

To obtain the wages for each value of \( \varphi \), we start with the value \( z \) at \( \varphi^* \). Then, we add \( \partial w(\varphi)/\partial \varphi \times \text{step size of} \ \varphi \) to the previous wage, where \( \partial w(\varphi)/\partial \varphi \) is given by (34). Labor input per firm is calculated using equation (33).

Given wages \( w(\varphi) \) and labor inputs \( l(\varphi) \) the next steps within the same loop are to recalculate vacancies \( v(\varphi) \) according to (31) and labor inputs \( l(\varphi) \) according to (33). We then calculate the sum over the changes in \( v(\varphi) \) from the previous and current calculation in the loop. If this change is positive, we increase every element in \( v(\varphi) \) by multiplying the old values by 0.9999, and otherwise by 1.0001. We repeat this inner loop until the sum of the changes of \( v(\varphi) \) is smaller than 0.01. We then recalculate the integral \( \int_{\varphi}^{\bar{\varphi}} \frac{v(\widetilde{\varphi})}{1-F(\varphi^*)} d\Gamma(\widetilde{\varphi}) \) for each value of \( \varphi \).

Given the values of \( \tilde{v}, v(\varphi), w(\varphi), \) and \( l(\varphi) \), we construct a matrix of size grid size \( \times \) (number of countries), where we calculate for each value of \( \varphi \) the (potential) profits if the firm does not export, exports to one country, exports to two countries, and so on, up to the maximum number of trading partners. Profits are given by equation (37). Given this matrix we next construct a vector that contains the number of countries that a firm with productivity \( \varphi \) should export in order to maximize profits. The zero cutoff productivity \( \varphi^* \) is given by the value of \( \varphi \) where total profits are equal to zero and where it is profit maximizing for a firm to serve only the home market.

\[12\]We solve our model using Matlab Release R2009b. The m-file is available upon request from the authors.
After a first initialization of a chosen value of $M$, we calculate the free entry condition as given in equation (40). If this value is negative, we reduce the number of firms $M$ by 0.1%; otherwise we increase it by 0.1%. We then repeat the whole process with the new value of $M$ until $M$ converges.$^{13}$

For the simulations we have chosen the following parameter values, $\chi = 0.05$, $\eta = 0.01$, $\delta = 0.02$, $\phi = 0.02$, $\rho = 0.75$, $\tau = 1.8$, $c = 1000000$, $\alpha = 5$, $f = 0.0002$, $f_x = 35f$, $f_e = 10$, $\gamma = 3.4$, $\varphi = 10$, $\overline{\varphi} = 100$ and $z = 1$. For the case with trade we assume 99 trading partners, i.e., 100 countries.$^{14}$

### 5.5.2 Results

Throughout this section we focus on two scenarios: A world where the country is in autarky and a world where there are 99 symmetric trading partners.$^{15}$

In Figure 6 we plot the number of vacancies created (left panel) and the number of export markets served by a firm with productivity $\varphi$ (right panel). In line with Proposition 1 the number of export markets served is an increasing function of productivity. We calibrated the model such that no firm is willing to export to all foreign markets. Firms with the highest productivities enter 57 out of the 99 markets. Like in the model with fixed vacancies the level of productivity $\varphi^*$ where firms still make positive profits is higher in the open economy than in autarky.

With trade the number of vacancies per firm is lower than in autarky for firms with a low productivity but higher for firms with a high productivity. Additionally, the number of vacancies are increasing in productivity in both scenarios. More importantly, the number of vacancies jumps up at each export cutoff because firms increase their labor input in response to additional demand from abroad.

Figure 7 plots labor inputs (left panel) and outputs (right panel) per firm. The pattern of vacancies translates into labor input and output pattern. In the open econ-

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$^{13}$Our convergence criterion is $\left| \left( \int_{\varphi^*}^{\overline{\varphi}} \Pi_{\max}(\varphi) \gamma(\varphi) \, d\varphi \right) - f_e \right| < 0.01$.

$^{14}$The grid size is chosen to be 1000. However, results do not depend on the chosen grid size.

$^{15}$The number of (potential) trading partners is not crucial for the basic qualitative results.
Figure 6: Vacancies and number of countries served in autarky and in an open economy with endogenous vacancies.

...
Figure 7: Firm size (labor input and output) in autarky and in an open economy with endogenous vacancies

The domestic price of the least productive exporter that serves more than one country is slightly lower than the price charged by the least productive exporter that only serves one foreign market. However, the price is still higher than the domestic price of the firm that only serves the local market. These results are similar to our results shown in Figure 4b.

Quantities are just the reverse image of prices charged in the domestic market. The right panel shows that the domestically sold quantities are much higher under autarky than in an open economy, specifically for very productive firms. The quantity of the least productive firm is higher than the quantity of the least productive firm serving in addition to the domestic market one foreign market. Hence, the results that we derived in Proposition 3 survive under endogenous vacancy creation.

Figure 9 shows profits as a function of productivity. In both scenarios, autarky and trade, profits are increasing in productivity. There are no jumps in the profit function,

\footnote{We set the number of (potential) trading partners large enough so that even the most productive firm does not serve all foreign markets. If we would allow a firm to serve all export markets, this firm could only expand by lowering prices. This would be reflected by a fall of the price line at the right end.}
Figure 8: Domestic quantities and prices in autarky and in an open economy with endogenous vacancies

because the extra revenues from exporting are used to pay for the foreign market entry costs. This is due to the definition of the export cutoff, where the least productive firm entering \( j \) markets has to be indifferent between entering \( j \) markets or only serving \( j - 1 \) markets.

If we compare the profits of firms in autarky and in an open economy, we see that the profit function under trade is much steeper than under autarky. The reason is that by serving more than one market, a firm can demand higher prices in every market and therefore can generate higher profits with the same output. Furthermore, like in Melitz (2003) there are some firms with low productivities that make lower profits in an open economy compared to autarky, because the increased competition in the labor market reduces the labor input of low productivity firms and thus the output necessary to generate higher profits.

In Figure 10 we plot wages as a function of productivity (left panel) and the wage distribution (right panel). Wages are an increasing function of productivity under both, autarky and trade. The following three observations are interesting: (i) The wage distribution starts at a lower productivity values in autarky than in an open economy. This reflects the fact that only more productive firms can survive in an open
Figure 9: Profits as a function of productivity in autarky and in an open economy with endogenous vacancy creation.

(ii) Wages are at least as high as unemployment benefits \( z \). (iii) The wage function is much steeper in an open economy, because exporting generates higher profits and opens up the opportunity for firms to pay higher wages.

We can also compare the wage distribution in autarky and in an open economy. The right panel of Figure 10 shows that in both situations the lowest wage is given by \( z \). Since wages increase at exporting firms, opening up to trade leads to more wage dispersion as predicted in Proposition 4. Hence, allowing for vacancy creation does not lead to different conclusions regarding the effects of trade on the wage distribution. Note, that with endogenous vacancy creation it still holds that in an open economy the number of firms is higher and the unemployment rate lower compared to autarky.

5.5.3 Convex vacancy costs and concave hiring costs

Convex vacancy costs are crucial for our results. The empirical evidence on the shape of the vacancy cost function is small. Abowd and Kramarz (2003) and Kramarz and Michaud (2010) have shown that the shape of the hiring cost function for French firms

\[ \text{The effect is very small, though. Hence, it can not be seen in the figure.} \]
Figure 10: Wages and wage distributions in autarky and in an open economy with endogenous vacancy creation.

is mildly concave, while Blatter, Mühlemann and Schenker (2009) have shown that the shape of the hiring cost function for Swiss firms is convex. In this section we show that a convex vacancy cost function is consistent with a concave and a convex hiring cost function. Hiring cost functions have the same shape as the vacancy cost functions if the hiring rate $h(v)$ per vacancy is the same for all firms. However, the hiring rate per vacancy is increasing in the wage because job offers made by high wage firms are accepted by more employed workers. This property holds in the Burdett-Mortensen model like in any monopsony wage model as shown by Manning (2006).\textsuperscript{18} It can also be seen by looking at the equation for the hiring rate per vacancy given by,

$$h(v) = \eta \left[ u + (1 - u) G(w) \right].$$

In addition the number of vacancies are an increasing function of the wage paid by firms, i.e.,

$$\frac{\partial v(w)}{\partial w} > 0.$$

\textsuperscript{18}Using firm level data from the Labour Turnover Survey in the UK, Manning (2006) shows that there are increasing marginal costs of recruitment, i.e., that the vacancy cost function is convex.
Thus, the total number of workers hired $H = h(v)v$ increase with the wage for two reasons: (i) the number of vacancies created increase with the wage and (ii) the hiring rate per vacancy increases with the wage.

Now consider the shape of the hiring cost function $K(H)$ given any convex vacancy cost function $c(v)$ with $c'_v(v) > 0$ and $c''_{vv}(v) > 0$. Using the inverse function of $H = h(v)v$ and $v(w)$, the first derivative of the hiring cost function is given by,

$$\frac{\partial K(H)}{\partial H} = c'_v(v) \frac{\partial v}{\partial H} = c'_v(v) \frac{1}{h(v) + v} \frac{\partial h(v)}{\partial w} \frac{\partial w(v)}{\partial v} > 0,$$

where the inequality follows from,

$$\frac{\partial h(v)}{\partial w} = (1 - u) g'(w) > 0 \quad \text{and} \quad \frac{\partial w(v)}{\partial v} > 0.$$

The second derivative that determines the shape of the hiring cost function is given by,

$$\frac{\partial^2 K(H)}{\partial H^2} = c''_{vv}(v) \left( \frac{\partial v}{\partial H} \right)^2 - c'_v(v) \frac{2}{\partial w} \frac{\partial h(v)}{\partial v} \frac{\partial w(v)}{\partial v} + v \left( \frac{\partial^2 h(v)}{\partial v^2} \right)^2 + v \frac{\partial h(v)}{\partial w} \frac{\partial^2 w(v)}{\partial v^2} \frac{\partial v}{\partial H},$$

where

$$\frac{\partial^2 h(v)}{\partial v^2} = (1 - u) g''(w) \geq 0 \quad \text{and} \quad \frac{\partial^2 w(v)}{\partial v^2} \geq 0.$$  

Thus, a convex vacancy cost function implies a concave hiring cost function, if and only if

$$c''_{vv}(v) < c'_v(v) \left( \frac{2}{\partial w} \frac{\partial h(v)}{\partial v} \frac{\partial w(v)}{\partial v} + v \left( \frac{\partial^2 h(v)}{\partial v^2} \right)^2 + v \frac{\partial h(v)}{\partial w} \frac{\partial^2 w(v)}{\partial v^2} \right) \frac{\partial v}{\partial H},$$

which is feasible since $\partial h(v)/\partial w > 0$ and $\partial w(v)/\partial v > 0$. Thus, a convex vacancy cost function is consistent with a concave hiring cost function as found by Abowd and Kramarz (2003) and Kramarz and Michaud (2010) for French firms as well as a convex hiring cost function as found by Blatter, Mühlemann and Schenker (2009) for Swiss firms. Our simulations also provide an example that a convex vacancy cost function
leads to a concave hiring cost function as shown in the following Figure.\textsuperscript{19}

6 Conclusions

The implications of trade liberalization on firms' behavior is one of the most heavily discussed consequences of increasing globalization. We show that capacity constraints change firms' responses to trade liberalization compared to models with perfect labor markets or with imperfect labor markets without capacity constraining effects. With capacity constraining labor market frictions not all firms will serve all export markets, even when export markets are similar. Rather the number of export markets served by a firm is increasing in its productivity or product quality. Given the capacity constraints that firms face if they want to recruit more workers in their domestic country, an obvious extension of our model is to allow for foreign direct investment. This would allow firms to relax their capacity constraints by recruiting and producing in a foreign country.

\textsuperscript{19}The parametrization is as follows: $\chi = 0.02$, $\eta = 0.9$, $\delta = 0.02$, $\phi = 0.02$, $\rho = 0.75$, $c = 500$, $\alpha = 1.01$, $f = 0.0001$, $f_e = 5$, $\gamma = 3.2$, $\varphi = 30$, $\overline{\varphi} = 100$ and $z = 1$. We only focus on the case of autarky here.
Concerning trade liberalization and labor market outcomes we find that unemployment falls and wage dispersion increases with trade liberalization. Opening up to trade increases expected profits, triggers firm entry and reduces unemployment. Increased profits of exporting firms also increases wages at the upper end of the wage distribution. Wages at the lower end of the wage distribution are pinned down by workers’ reservation wage which equals unemployment benefits.

References


Appendix A: Wages offers

Assumption 1 ensures that no mass point exist

We first show that a mass point can only exist at the lower end of the wage distribution. Suppose that firms with different productivities \( \varphi > \varphi' \) pay the same wage \( \tilde{w} \). If \( \tilde{w} \) is optimal for \( \varphi' \), i.e., \( \Pi'_{\tilde{w}} (\varphi', \tilde{w}) = 0 \), then,

\[
\delta \frac{\partial \Pi (\varphi, \tilde{w})}{\partial \tilde{w}} = \left[ \varphi' \rho l (\tilde{w})^{(\rho-1)} - \tilde{w} \right] \frac{\partial l (\tilde{w})}{\partial \tilde{w}} - l (\tilde{w}) > 0,
\]

which implies that the firm with productivity \( \varphi \) will optimally increase its wage, i.e., \( w (\varphi) > \tilde{w} \). If \( \tilde{w} \) is optimal for \( \varphi \), i.e., \( \Pi_{\tilde{w}} (\varphi, \tilde{w}) = 0 \), then again the first order condition implies \( \Pi'_{\tilde{w}} (\varphi', \tilde{w}) < 0 \). Thus, the firm with productivity \( \varphi' \) will choose a wage \( w (\varphi') < \tilde{w} \) as long as the wage \( \tilde{w} \) is above the reservation wage, i.e., \( \tilde{w} > z \).

We also know that the reservation wage is paid by the firm with the cutoff productivity, i.e.,

\[
\Pi (\varphi^*, z) = 0 \implies [\varphi^* l (z)]^\rho = z l (z) + f.
\]

To derive assumption 1 we want to get a condition that ensures \( \Pi'_{\tilde{w}} (\varphi', z) \geq 0 \) for all \( \varphi' \in [\varphi^*, \varphi] \). Using the cutoff condition to substitute \( l (z)^\rho \) in the first order condition of the firms paying the reservation wage gives,

\[
\rho \left[ \frac{\varphi'}{\varphi^*} \right]^\rho [z l (z) + f] \geq z l (z) + l (z)^2 \left[ \frac{\partial l (z)}{\partial z} \right]^{-1}.
\]

Rearranging implies,

\[
\rho f \geq \left[ \frac{\varphi^*}{\varphi'} \right]^\rho z l (z) - \rho z l (z) + \left[ \frac{\varphi^*}{\varphi'} \right]^\rho l (z)^2 \left[ \frac{\partial l (z)}{\partial z} \right]^{-1},
\]

\[
> (1 - \rho) z l (z) = (1 - \rho) z \frac{\eta \Pi (\varphi + \phi)}{[\varphi + \phi + \eta M v]^2}.
\]

Note, that the number of active firms \( M < 1 \), since the number of workers is normalized to one. Thus, letting \( M \to 1 \) implies that inequality (17) is sufficient to ensure that no mass point exists, i.e., to ensure \( \Pi'_{\tilde{w}} (\varphi', z) \geq 0 \) for all \( \varphi' \in [\varphi^*, \varphi] \).

Vacancy posting if a mass point exist
Suppose that some firms have productivity \( \varphi' \in [\varphi^*, \tilde{\varphi}) \) such that \( \Pi'(w, z) = 0 \), \( \Pi'_w (\varphi', z) < 0 \) and \( \Pi (\varphi^*, z) = 0 \). Thus, firms with productivity \( \varphi' \) would like to employ less workers in order to ensure \( \Pi'_w (\varphi', z) = 0 \). In order to do so they can open less vacancies. The number of workers that a firm with wage \( z \) and \( \beta \) vacancies employs is given by,
\[
l(z, \beta) = \frac{\eta \beta (\varphi + \phi)}{[\varphi + \phi + \lambda (M) \nu(z)] [\varphi + \phi + \lambda (M)]},
\]
where \( \nu(z) = [\Gamma (\tilde{\varphi}) - \Gamma (\varphi^*)] / [1 - \Gamma (\varphi^*)] \) is the mass of firms offering a wage \( z \).

The first order condition is then given by,
\[
\delta \frac{\partial \Pi (\varphi', z, \beta)}{\partial \beta} = \rho \beta^{-1} \left[ \varphi' l(z, 1) \right] - zl(z, 1) = 0,
\]
which implies that firms with productivity \( \varphi' \) open \( \beta (\varphi') \tilde{\nu} \) vacancies, with \( \beta (\varphi') \in (0, 1) \). The fact that firms with productivity \( \varphi' \in [\varphi^*, \tilde{\varphi}) \) open less vacancies changes the firm contact rate to,
\[
\lambda (M) = \eta M \frac{\tilde{\nu}}{1 - \Gamma (\varphi^*)} \left[ \int_{\varphi^*}^{\tilde{\varphi}} \beta (\varphi') d\Gamma (\varphi') + 1 - \Gamma (\tilde{\varphi}) \right].
\]
Given \( \lambda (M) \) the analysis then follows the general case with endogenous vacancy creation in section 5.

**Appendix B: Derivation of the wage function \( w (\varphi) \)**

The first order condition implies,
\[
1 = \left[ \varphi^p \rho l (w (\varphi))^{(p-1)} - w (\varphi) \right] \frac{\partial l (w (\varphi))}{\partial w (\varphi)} \frac{1}{l (w (\varphi))}.
\]
Using the fact that wages increase with productivity gives,
\[
\frac{\partial w (\varphi)}{\partial \varphi} = \left[ \varphi^p \rho l (\varphi)^{(p-1)} - w (\varphi) \right] \frac{\partial l (\varphi)}{\partial \varphi} \frac{1}{l (\varphi)}.
\]
Define,
\[
T (\varphi) = \ln l (\varphi) \quad \text{and} \quad T' (\varphi) = \frac{\partial l (\varphi)}{\partial \varphi} \frac{1}{l (\varphi)}.
\]
Substitution simplifies the above differential equation to,
\[
\frac{\partial w}{\partial \varphi} + w(\varphi) T'(\varphi) = \rho \varphi^\rho \left[ e^{T(\varphi)} \right]^{\rho-1} T'(\varphi).
\]
Any solution to this differential equation has to satisfy,
\[
w(\varphi) e^{T(\varphi)} = \int_{\varphi'}^{\varphi} \rho \left[ \tilde{\varphi} e^{T(\tilde{\varphi})} \right]^\rho T'(\tilde{\varphi}) d\tilde{\varphi} + A,
\]
where \( A \) is the constant of integration. Note that,
\[
\frac{d}{d\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho = \rho \varphi^\rho \left[ e^{T(\varphi)} \right]^\rho T'(\varphi) + \rho \varphi^{\rho-1} \left[ e^{T(\varphi)} \right]^\rho.
\]
The integral can thus be written as,
\[
\int_{\varphi'}^{\varphi} \rho \left[ \tilde{\varphi} e^{T(\tilde{\varphi})} \right]^\rho T'(\tilde{\varphi}) d\tilde{\varphi} = \int_{\varphi'}^{\varphi} \rho \tilde{\varphi}^\rho \left[ e^{T(\tilde{\varphi})} \right]^\rho d\tilde{\varphi} - \rho \tilde{\varphi}^{\rho-1} \left[ e^{T(\tilde{\varphi})} \right]^\rho d\tilde{\varphi}
\]
\[
= \left[ \varphi e^{T(\varphi)} \right]^\rho - \left[ \varphi^* e^{T(\varphi^*)} \right]^\rho - \int_{\varphi'}^{\varphi} \rho \tilde{\varphi}^{\rho-1} \left[ e^{T(\tilde{\varphi})} \right]^\rho d\tilde{\varphi}.
\]
Substituting into the wage equation (43) gives,
\[
w(\varphi) e^{T(\varphi)} = \left[ \varphi e^{T(\varphi)} \right]^\rho - \left[ \varphi^* e^{T(\varphi^*)} \right]^\rho - \int_{\varphi'}^{\varphi} \rho \tilde{\varphi}^{\rho-1} \left[ e^{T(\tilde{\varphi})} \right]^\rho d\tilde{\varphi} + A
\]
\[
+ z \frac{e^{T(\varphi^*)}}{e^{T(\varphi)}}
\]
since,
\[
w(\varphi^*) e^{T(\varphi^*)} = ze^{T(\varphi^*)} = A.
\]
Substituting \( e^{T(\varphi)} = l(\varphi w(\varphi)) \) and \( z \) by using the zero cutoff condition (8) gives the wage equation (19).

**Appendix C: Equilibrium in autarky**

Applying the implicit function theorem to the free entry condition (20) implies,
\[
\frac{d\varphi^*}{dM} = -\frac{\int_{\varphi'}^{\varphi} \rho^2 \left[ \tilde{\varphi} l(\tilde{\varphi}) \right]^{\rho-1} \frac{\partial l(\tilde{\varphi})}{\partial \varphi^*} d\tilde{\varphi}}{\int_{\varphi'}^{\varphi} \rho^2 \left[ \tilde{\varphi} l(\tilde{\varphi}) \right]^{\rho-1} \frac{\partial l(\tilde{\varphi})}{\partial M} d\tilde{\varphi}} < 0,
\]
where the inequality follows because $\partial l(w(\tilde{\varphi})) / \partial \varphi^* < 0$ and $\partial l(w(\tilde{\varphi})) / \partial M < 0$. Thus, the free entry condition defines a decreasing relation between the zero cutoff productivity $\varphi^*$ and the number of active firms $M$ in the market.

Applying the implicit function theorem to the zero profit condition (21) implies,

$$\frac{d\varphi^*}{dM} = \frac{\rho[\varphi^*]^{\rho-1} [l(z)]^{\rho-1} - z \left( \frac{2\eta\pi}{\pi + \phi + \eta Mv} \right) l(z)}{\rho[\varphi^*]^{\rho-1} [l(z)]^{\rho}} > 0,$$

where Assumption 1, i.e., $\Pi^*_w(\varphi^*, z) \geq 0 \implies \rho[\varphi^*]^{\rho} [l(z)]^{(\rho-1)} > z$, ensures an increasing relation between the zero cutoff productivity $\varphi^*$ and the number of active firms $M$ in the market.

An equilibrium only exists if unemployment benefits $z$ and the fixed costs $f$ are low enough.

**Appendix D: The open economy**

*Quantities sold in the domestic and each export market*

An exporting firm that decided to serve $j$ foreign countries maximizes its profits by equalizing marginal revenues across markets. Revenues of an exporting firm are given by,

$$R(\varphi) = p_d(\varphi) \varphi q_d(\varphi) + j \frac{p_x(\varphi)}{\tau} \varphi q_x(\varphi)$$

$$= p_d(\varphi) \varphi \left[ q(\varphi) - jq_x(\varphi) \right] + j \frac{p_x(\varphi)}{\tau} \varphi q_x(\varphi)$$

$$= \varphi^{\rho} \left[ q(\varphi) - jq_x(\varphi) \right]^{\rho} + j \left[ \frac{q_x(\varphi)}{\tau} \right]^{\rho}.$$

By choosing its domestic and export sells according to equalization of marginal rev-
\[
\frac{\partial R(\varphi)}{\partial q_x(\varphi)} = 0, \\
\rho_j [q(\varphi) - jq_x(\varphi)]^{\rho - 1} = \rho_j \frac{1}{\tau} \left[ \frac{q_x(\varphi)}{\tau} \right]^{\rho - 1}, \\
q(\varphi) - jq_x(\varphi) = \frac{1}{\tau^{\rho/(\rho - 1)}} q_x(\varphi), \\
\tau^{\rho/(\rho - 1)} q(\varphi) = [1 + j\tau^{\rho/(\rho - 1)}] q_x(\varphi).
\]

Rearranging and using the fact that \(q_{d}(\varphi) = q(\varphi) - jq_x(\varphi)\) implies equations (22) and (23). The revenue of an exporting firm is, therefore, given by,

\[
R(\varphi) = \varphi^\rho \left[ q(\varphi) - jq(\varphi) \right]^{\tau^{\rho/(\rho - 1)}} + j \left[ \frac{q(\varphi)}{\tau} \right]^{\tau^{\rho/(\rho - 1)}} \]

\[
= [1 + j\tau^{\rho/(\rho - 1)}] \left[ \frac{\varphi q(\varphi)}{1 + j\tau^{\rho/(\rho - 1)}} \right]^\rho \\
= [1 + j\tau^{\rho/(\rho - 1)}]^{(1 - \rho)} [\varphi q(\varphi)]^\rho.
\]

**Proof of Proposition 1: Export cutoffs**

The export cutoff productivity \(\varphi_x^n\) is defined by \(\delta \Pi_{d+j} (\varphi_x^n) = \delta \Pi_{d+j-1} (\varphi_x^n)\), where

\[
\delta \Pi_{d+j}(\varphi) = \left[ 1 + j\tau^{\rho/(\rho - 1)} \right]^{(1 - \rho)} [\varphi l (w(\varphi))]^{\rho} - w(\varphi) l (w(\varphi)) - f - jf_x.
\]

Since profit maximization implies that the wage is continuous at \(\varphi_x^n\), and since the same wage implies that the number of workers employed by both type of firms are identical and given by \(l (w(\varphi_x^n))\) we may write,

\[
\left[ 1 + j\tau^{\rho/(\rho - 1)} \right]^{(1 - \rho)} [\varphi_x^l (w(\varphi_x^n))]^{\rho} - w(\varphi_x^n) l (w(\varphi_x^n)) - f - jf_x \\
= \left[ 1 + (j - 1)\tau^{\rho/(\rho - 1)} \right]^{(1 - \rho)} [\varphi_x^l (w(\varphi_x^n))]^{\rho} - w(\varphi_x^n) l (w(\varphi_x^n)) - f - (j - 1) f_x.
\]

Thus, the export cutoff condition (26) can be derived,

\[
[\varphi_x^l (w(\varphi_x^n))]^{\rho} = \frac{f_x}{\left[ 1 + j\tau^{\rho/(\rho - 1)} \right]^{(1 - \rho)} - \left[ 1 + (j - 1)\tau^{\rho/(\rho - 1)} \right]^{(1 - \rho)}}.
\]
where the last inequality follows from Jensen’s inequality, i.e.,

\[
\frac{1}{2} \left[ 1 + (j - 2) \frac{\rho}{\tau - j} \right]^{(1-\rho)} + \frac{1}{2} \left[ 1 + j \frac{\rho}{\tau - j} \right]^{(1-\rho)} < \left[ 1 + \frac{1}{2} (j - 2) + \frac{1}{2} j \right] \frac{\rho}{\tau - j} \left[ j \frac{\rho}{\tau - j} \right]^{(1-\rho)} = \left[ 1 + (j - 1) \frac{\rho}{\tau - j} \right]^{(1-\rho)}.
\]

**Wages in an open economy**

The wage equation for exporting firms follows from the first order condition and the equilibrium condition (18), leading to the following differential equation,

\[
\frac{\partial w(\varphi)}{\partial \varphi} + w(\varphi) T'(\varphi) = \left[ 1 + j \frac{\rho}{\tau - j} \right]^{(1-\rho)} \rho \varphi^{\rho} \left[ e^{T(\varphi)} \right]^{\rho-1} T'(\varphi),
\]

where \( T(\varphi) \) and \( T'(\varphi) \) are defined in Appendix B. The solution to this differential equation is obtained by following the same steps as in Appendix B, and given by,

\[
w(\varphi) e^{T(\varphi)} = \left[ 1 + j \frac{\rho}{\tau - j} \right]^{(1-\rho)} \left[ \varphi e^{T(\varphi)} \right]^{\rho} - \left[ \varphi_x^{j} e^{T(\varphi)^{j-1}} \right]^{\rho} - \int_{\varphi_x^{j-1}}^{\varphi_x^{j}} \frac{\rho}{\varphi} \left[ \tilde{\varphi} e^{T(\tilde{\varphi})} \right]^{\rho} d\tilde{\varphi} + A,
\]

where,

\[
A = w \left( \varphi_x^{j-1} \right) e^{T(\varphi_x^{j-1})}
\]

\[
= \left[ 1 + (j - 1) \frac{\rho}{\tau - j} \right]^{(1-\rho)} \left[ \varphi_x^{j-1} e^{T(\varphi_x^{j-1})} \right]^{\rho} - \left[ \varphi_x^{j-1} e^{T(\varphi_x^{j-1})} \right]^{\rho} - \int_{\varphi_x^{j-1}}^{\varphi_x^{j-1}} \frac{\rho}{\varphi} \left[ \tilde{\varphi} e^{T(\tilde{\varphi})} \right]^{\rho} d\tilde{\varphi} + w \left( \varphi_x^{j-1} \right) e^{T(\varphi_x^{j-1})},
\]

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and,
\[ w (\varphi^1_x) e^{T(\varphi^1_x)} = \left[ \varphi^1_x e^{T(\varphi^1_x)} \right]^\rho - \left[ \varphi^* e^{T(\varphi^*)} \right]^\rho - \int_{\varphi^*}^{\varphi^1_x} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho \, d\varphi + z e^{T(\varphi^*)}. \]

Since \( A \) depends on the wage payments of those firms with export cutoff productivities \( \varphi^j_x < \varphi \) we need to rewrite the wage equation as follows,
\[
w (\varphi) e^{T(\varphi)} = \left[ 1 + j \tau \frac{\varphi^T}{\varphi^j_x} \right]^{(1-\rho)} \left[ \varphi e^{T(\varphi)} \right]^\rho - \int_{\varphi^j_x}^{\varphi} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho \, d\varphi
\]
\[+ \left[ 1 + (j - 1) \tau \frac{\varphi^T}{\varphi^j_x} \right]^{(1-\rho)} \left[ \varphi^j_x e^{T(\varphi^j_x)} \right]^\rho - \int_{\varphi^j_x}^{\varphi^j_x - 1} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho \, d\varphi + \ldots
\]
\[+ \left[ \varphi^j_x e^{T(\varphi^j_x)} \right]^\rho - \left[ \varphi^* e^{T(\varphi^*)} \right]^\rho - \int_{\varphi^*}^{\varphi^j_x} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho \, d\varphi + ze^{T(\varphi^*)},
\]
or,
\[
w (\varphi) e^{T(\varphi)} = \left[ 1 + j \tau \frac{\varphi^T}{\varphi^j_x} \right]^{(1-\rho)} \left[ \varphi e^{T(\varphi)} \right]^\rho - \int_{\varphi^j_x}^{\varphi} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho \, d\varphi
\]
\[- \sum_{i=1}^{j} \left[ 1 + i \tau \frac{\varphi^T}{\varphi^i_x} \right]^{(1-\rho)} \left[ 1 + (i - 1) \tau \frac{\varphi^T}{\varphi^i_x} \right]^{(1-\rho)} \left[ \varphi^i_x e^{T(\varphi^i_x)} \right]^\rho
\]
\[- \sum_{i=2}^{j} \left[ 1 + (i - 1) \tau \frac{\varphi^T}{\varphi^i_x} \right]^{(1-\rho)} \int_{\varphi^i_x}^{\varphi^i_x - 1} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho \, d\varphi
\]
\[- \left[ \varphi^* e^{T(\varphi^*)} \right]^\rho + \int_{\varphi^*}^{\varphi^j_x} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho \, d\varphi \right] + ze^{T(\varphi^*)}.
\]

Substituting \( e^{T(\varphi)} = l \left( w (\varphi) \right), z \) by using the zero cutoff condition (21) and \( [\varphi^j_x l \left( w (\varphi^j_x) \right)]^\rho \) using the export cutoff condition (26) gives,
\[
w (\varphi) l \left( w (\varphi) \right) = \left[ 1 + j \tau \frac{\varphi^T}{\varphi^j_x} \right]^{(1-\rho)} \left[ \varphi l \left( w (\varphi) \right) \right]^\rho - j f_x
\]
\[- \left[ 1 + j \tau \frac{\varphi^T}{\varphi^j_x} \right]^{(1-\rho)} \int_{\varphi^j_x}^{\varphi} \frac{\rho}{\varphi} \left[ \varphi l \left( w (\varphi) \right) \right]^\rho \, d\varphi
\]
\[- \sum_{i=1}^{j} \left[ 1 + (i - 1) \tau \frac{\varphi^T}{\varphi^i_x} \right]^{(1-\rho)} \int_{\varphi^i_x}^{\varphi^i_x - 1} \frac{\rho}{\varphi} \left[ \varphi l \left( w (\varphi) \right) \right]^\rho \, d\varphi
\]
\[- \int_{\varphi^*}^{\varphi^j_x} \frac{\rho}{\varphi} \left[ \varphi l \left( w (\varphi) \right) \right]^\rho \, d\varphi - f.\]
The wage equation (25) follows immediately by defining $\varphi = \varphi^{j+1}_x$ and $\varphi^* = \varphi^0_x$.

**Average profits in an open economy**

Rearranging the wage equation (25) implies that profits of an exporting firm are given by,

$$\delta \Pi_{d+j} (\varphi) = \left[1 + j \tau \frac{\rho}{\rho - 1} \right] (1 - \rho) \left[ \frac{\varphi l (w (\varphi))}{\rho} - w (\varphi) l (w (\varphi)) - f - j f x \right]$$

$$= \sum_{i=1}^{j+1} \left[1 + (i - 1) \tau \frac{\rho}{\rho - 1} \right] (1 - \rho) \int_{\varphi^{i-1}}^{\varphi^i} \frac{\rho}{\varphi} \left[ \varphi l (w (\varphi)) \right] d\varphi,$$

where $\varphi^{j+1}_x = \varphi$ and $\varphi^0_x = \varphi^*$. Since free entry implies $f_e = \Pi_e (\varphi^*) = \int_{\varphi^*}^{\varphi} \Pi (\varphi) \gamma (\varphi) d\varphi$, integrating over all firms with productivity $\varphi \in [\varphi^*, \varphi]$ implies the free entry condition for an open economy as stated in equation (27).

**Proof of Proposition 2: Upward rotation of the free entry condition**

We need to show that for each $\varphi^* \in [0, \varphi]$ the number of active firms increases, i.e., $M_T > M_A$. Suppose the opposite, i.e., $M_T \leq M_A$. Thus, labor input for a firm with productivity $\varphi$ is given by $l_T (w (\varphi)) \geq l_A (w (\varphi))$ according to equation (16). Since $\left[1 + (i - 1) \tau \frac{\rho}{\rho - 1} \right] (1 - \rho) > 1$, it follows that $\Pi_e (\varphi^*)|_{Trade} > \Pi_e (\varphi^*)|_{Autarky} = f_e$. This contradicts, however, the free entry condition in an open economy. Thus, $M_T > M_A$.

The increase in the number of active firms $M$ increases the zero cutoff productivity $\varphi^*$. It is easy to verify from equation (16) that the size of all firms $l (w (\varphi))$ decreases.

**Appendix E, Proof of Proposition 4: Unemployment and wage dispersion in an open economy**

It follows from equation (12) that the unemployment rate decreases as the number of active firms $M$ increases in response to opening the economy for trade.

The increase in the number of active firms $M$ and the increase in the zero cutoff productivity $\varphi^*$ in response to opening up to trade implies that wages increase if the
position of a firm in the wage offer distribution, i.e., \( F(w) \), is kept constant. This can be proven by using equation (25), i.e.,

\[
    w(\varphi) l(w(\varphi)) = \left[ 1 + j \tau \frac{\varphi^*}{\varphi - 1} \right]^{(1-\rho)} \varphi^0 l(w(\varphi))^\rho - f - j f_x \\
    - \sum_{i=1}^{j+1} \left[ 1 + (i - 1) \tau \frac{\varphi^*}{\varphi - 1} \right]^{(1-\rho)} \int_{\varphi_{x-1}}^{\varphi_x} \rho \left[ \varphi l(w(\varphi)) \right]^\rho d\varphi,
\]

where \( \varphi_{x+1} = \varphi \) and \( \varphi_0^0 = \varphi^* \). First, keep \( l(w(\varphi)) \) constant and notice that for a given labor input \( l(w(\varphi)) \) wages increase when opening up to trade, because \( \left[ 1 + j \tau \frac{\varphi^*}{\varphi - 1} \right] > \left[ 1 + i \tau \frac{\varphi^*}{\varphi - 1} \right] \) for all \( i < j \). Second, an increase in \( M \) leads to a lower labor size \( l(w(\varphi)) \). This reduces the wage costs, i.e., (lhs) \( w(\varphi) l(w(\varphi)) \), more than the revenues, i.e., (rhs) \( \left[ 1 + j \tau \frac{\varphi^*}{\varphi - 1} \right]^{(1-\rho)} \varphi^0 l(w(\varphi))^\rho \), as can be seen from the FOC, i.e.,

\[
    \left[ 1 + j \tau \frac{\varphi^*}{\varphi - 1} \right]^{(1-\rho)} \rho \varphi^0 \left[ l(w(\varphi)) \right]^{\rho} - w(\varphi) l(\varphi) = \frac{\partial w(\varphi)}{\partial l(\varphi)} l(\varphi)^2 > 0.
\]

Thus, the wage must increase since a decrease in labor input \( l(w(\varphi)) \) decreases the lhs more than the rhs. Third, an increase in \( \varphi^* \) while keeping \( F(w) \) constant reduces the integral in the second line of the above equation and thereby increases the wage. Since the position of the wage distribution of the highest productivity firm, i.e., \( F(w(\varphi)) = 1 \), remains unchanged, it follows that the highest wage increases. Since the lowest wage equals the level of unemployment benefits \( z \) in the closed and open economy, it follows that wage dispersion \( w(\varphi) - z \) is higher in an open economy than in a closed economy.

**Appendix F: Vacancy creation condition for linear vacancy costs**

The optimality condition for vacancies (31) for \( \alpha = 1 \) and wages (32) imply the equality of marginal costs, i.e.,

\[
    cv(\varphi) \frac{\partial l(\varphi)}{\partial \varphi} \frac{\partial w(\varphi)}{\partial \varphi} = \frac{\partial w(\varphi)}{\partial \varphi} l(\varphi),
\]

\[ (46) \]
where we used \( \partial l(\varphi, v)/\partial v = l(\varphi, v)/v \). Similar to Appendix B we define,

\[
l(\varphi) = \eta v(\varphi)(\kappa + \phi)e^{T(\varphi)},
\]

where,

\[
T(\varphi) = -\log \left[ \kappa + \delta + \eta M \int_{\varphi}^{\tilde{\varphi}} \frac{v(\tilde{\varphi})}{1 - \Gamma(\varphi^*)} d\Gamma(\tilde{\varphi}) \right]^2,
\]

and,

\[
T'(\varphi) \equiv \frac{2\eta M v(\varphi) \gamma(\varphi)}{\kappa + \phi + \eta M \int_{\varphi}^{\tilde{\varphi}} \frac{v(\tilde{\varphi})}{1 - \Gamma(\varphi^*)} d\Gamma(\tilde{\varphi})} = \frac{\partial l(\varphi) \partial w(\varphi)}{l(\varphi)}. \]

Equation (46) can therefore be written as,

\[
\frac{\partial w(\varphi)}{\partial \varphi} = \frac{c}{\eta (\kappa + \phi)} \frac{T'(\varphi)}{e^{T(\varphi)}}.
\]

Integration gives,

\[
w(\varphi) = \frac{c}{\eta (\kappa + \phi)} \int_{\varphi}^{\varphi^*} \frac{T'(\tilde{\varphi})}{e^{T(\tilde{\varphi})}} d\tilde{\varphi} + A = \frac{c}{\eta (\kappa + \phi)} \left[ e^{-T(\varphi^*)} - e^{-T(\varphi)} \right] + A \]

\[
= c \left[ \frac{v(\varphi^*)}{l(\varphi^*)} - \frac{v(\varphi)}{l(\varphi)} \right] + z,
\]

where \( A = z \) follows from \( w(\varphi^*) = z \). Substituting the wage \( w(\varphi) \) into the optimality condition for vacancies (31) for \( \alpha = 1 \) gives the stated result.